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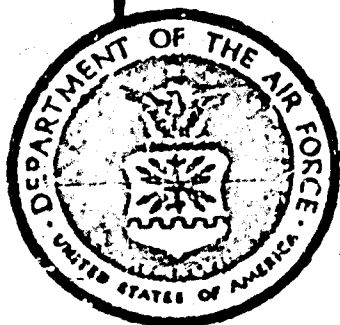
M. E. Serevryakov

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M. E. SEREZHAKOV

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CHAPTER III - CALCULATING THE HEAT LOST TO THE
WALLS DURING BURNING OF POWDER IN A CLOSED CHAMBER

When powder is burned in a closed chamber (in a bomb), a portion of the heat energy is lost on heating the walls of the bomb. As a result, the pressure p_m developed by the gases is somewhat lower than the theoretical pressure; the latter would be obtained if all the thermal energy emitted during the combustion of the given powder were utilized to increase the pressure of the gases.

This loss of heat depends on a number of loading factors.

Experiments conducted by professor S.P. Vukolov in the Naval Technical Research Laboratory back in 1895 and 1896 had shown that at $\Delta = 0.20$ the pressures p_m developed in a standard bomb and in a bomb whose interior surface was lined with a thin layer of nonconductive mica were different. The resultant pressures were: 2033 kg/cm² in the bomb without the mica layer and 2202 kg/cm² in the bomb containing the mica, the difference - 169 kg/cm² - comprises about 8%.

It was stated previously (in the theory of powder combustion), in the discussion of problems relating to powder ignition in a gun, that the contact of the powder grains with the cool surface of the chamber slows down the ignition process.

The following simple experiment in the open air will show this to be true. If a strip of powder is clamped vertically in a locksmith's metal vise and ignited from the top, the process of burning will be arrested upon reaching the vise. The strip will be extinguished because a considerable portion of the heat is taken up by the cold metal.

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It is known that a nonsimultaneous ignition distorts the initial shape of the grain and causes deviations from the ideal law of combustion. Consequently, the transfer of heat to the walls should have an effect not only on the magnitude of pressure, but also on the character of its development.

By disregarding losses in a bomb due to heat transfer we commit an error in the determination p_m and, consequently, an error in the magnitude of energy f and covolume α determined on the basis of experiments with a manometric bomb.

In view of this heat transfer, Nobel's formula will hold true for pressures above 1000 kg/cm^2 . At lower pressures, corresponding to charging densities of $\Delta < 0.10$, the points in the diagram $\frac{p_m}{\Delta}$ versus p_m will fall below the straight line relationship expressed by the known equation $\frac{p_m}{\Delta} = f + \alpha p_m$.

Systematic experiments conducted by assistant A.I. Kokhanov in 1933, at charging densities of from 0.015 to 0.20, have shown that a hyperbolic curve abc , approaching asymptotically the straight line de (fig. 16), is obtained in the system of coordinates $\frac{p_m}{\Delta}$, p_m instead of a straight line. The smaller the value of Δ , the greater is the deviation from the theoretical relationship.

The results obtained in determining the powder energy f and covolume α may therefore differ, depending on the charging densities at which the tests were conducted. The higher the value of Δ , the greater will be the magnitude of f and the smaller the covolume α (points b and c). And, conversely, small values of Δ should produce a small energy and a large covolume (points a and b).

Our experiments have also disclosed the following.

If a powder of the same composition but of different thickness is burned in a bomb at the same value of Δ , the maximum pressure p_m will be the lower the thicker the powder. Therefore, the energy produced by thick powders, determined without taking the heat losses into consideration, will also be the smaller, the thicker the powder. This can be explained by the fact that a thick powder burns longer and, consequently, a larger portion of the heat is transmitted to the walls of the bomb.

If an identical powder is burned at the same Δ in bombs of different sizes, the value of p_m will be higher in the larger bomb, because the surface per unit weight of powder charge is smaller in the larger bomb, and hence the heat losses will be smaller in it.

All of these facts confirm the presence of cooling through the walls of the bomb. The considerable number of tests conducted by various investigators made it possible to determine quantitatively the corrections to be introduced in the charging of powders of various thickness under different conditions of loading, in order to obtain pressures corrected for heat transfer. Although these corrections are not final, nevertheless their introduction served to explain the phenomena discussed above. Miura's correction may be considered to be the best rounded among corrections of this type.

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Fig. 16 - $\frac{p_{m1}}{\Delta}$ as a Function of p_{m1} , According to Kokhanov.

According to Miuraur the heat loss through transfer is proportional to the number of collisions between the powder gas molecules and the walls of the bomb, and this number is in turn proportional to the surface of the bomb S_f , pressure p and time t . For a fluctuating pressure, the loss is proportional to S_f and $\int p dt$. Also, because $\int p dt$ does not depend on Δ , the loss of heat ΔQ through the surface of walls S_f is constant for any powder charge ω or Δ . This was confirmed by Miuraur who conducted tests at different values of Δ . Inasmuch as the total quantity of the heat evolved is proportional to the weight of the charge ω , the relative loss $\frac{\Delta Q}{Q}$ is inversely proportional to ω or Δ . Consequently, $\frac{\Delta Q}{Q}$ is proportional to $\frac{S_f}{\omega} \int p dt$.

The magnitude of maximal pressure at a given value of Δ is governed by the volume of the bomb(*). Actually, an n% pressure

(*) Consequently, magnitudes f and α will also depend on the volume of the bomb in which the combustion takes place, while the magnitude of $\int p dt$ should not change because of cooling through the walls of the bomb.

drop entails a corresponding prolongation of the period of burning; the curve p, t becomes distorted, but the area $\int p dt$ remains unchanged.

All this was checked and verified by our tests with bombs of different sizes. The following important deduction can be reached from the above: the rate of burning $u_1 = \frac{e_1}{\int_0^1 p dt}$ can be determined with the same degree of accuracy in both large and small bombs, regardless of the heat lost through the walls.

Experiments for calculating heat losses. For quantitative determination of heat losses Miuraur conducted experiments at $\Delta = 0.20$ in a bomb measuring 150 cm^3 by volume. In one case the powder was burned under normal conditions, and in another a steel, trough-shaped insert with projections or ridges was inserted into the bomb in such a manner that its exterior surface did not come in contact with the surface of the bomb (fig. 17).

In the first instance, when the cooled surface S_1 was equal to the surface of the bomb S_6 , the obtained pressure was p_1 ; in the second case, when the cooled surface S_2 was larger, i.e., when it consisted of the surface of the bomb S_6 and the surface of the insert $S_{BKЛ}$ ($S_2 = S_6 + S_{BKЛ}$), the resultant pressure p_2 was lower. The pressure difference $\Delta p = p_1 - p_2$ resulted in consequence of the surface area difference $S_2 - S_1 = S_{BKЛ}$.

In order to determine the pressure in the absence of cooling, calculations were made of such an increment $\Delta p'$, which corresponded to a change in the surface area ΔS equal to the surface area of the bomb S_6 :

$$\frac{\Delta p'}{\Delta p} = \frac{S_6}{S_{BKЛ}} \quad \text{or} \quad \Delta p' = \Delta p \frac{S_6}{S_{BKЛ}}$$

In this manner, pressure $p_1 + \Delta p'$ would correspond to the surface $S = 0$, i.e., it would correspond to the condition in which losses due to heat transfer were absent.

Knowing p_1 and $\Delta p'$, Miuraur determined the relative correction for heat transfer $\frac{\Delta p'}{p} = \frac{\Delta p_m}{p}$, which was equal to $\frac{\Delta T}{T_1}$ in a constant volume.

Such tests conducted with a large number of powders of varied thicknesses and properties established the significant dependence of pressure loss $\frac{\Delta p_m}{p_m}$ on the time of powder burning at $\Delta = 0.20$, the magnitude S_g/ω in these tests being equal to $7.774 \text{ cm}^2/\text{g} = 77.74 \text{ dm}^2/\text{kg}$.

The experiments were conducted with cylindrical crushers. Very strong igniters of gunpowder were used to determine the actual powder burning time t_k . The igniters developed a pressure of $p_n \approx 250 \text{ kg/cm}^2$, which provided the crusher with small residual compression.

The obtained data was plotted, and the curve $\frac{\Delta p_m}{p_m} \%$, or $\frac{\Delta T}{T_1} \%$ as a function of the time of burning t_k , was termed "curve C" (fig. 18).

In order to determine the losses due to heat transfer under conditions other than those discussed, the powder must first be tested at $\Delta = 0.20$ and the time of burning t_k found, and the magnitude $C_M = \frac{\Delta p_m}{p_m}$ then determined from curve C. Losses under other conditions (in a different bomb and at a different density of loading), can be found by means of the following formula:

$$\frac{\Delta p_m}{p_m} = \frac{\Delta T}{T_1} = \frac{C_M \%}{7.774} \frac{S_g}{\omega} = \frac{C_M \%}{7.774} \frac{S_g}{W_0} \frac{1}{\Delta}. \quad (17)$$

C_M depends on the thickness and nature of the powder, while S_g/W_0 characterizes the relative surface of the bomb (exposed surface of

bomb) and can be calculated as the exposed surface of a uniform cylinder ($2c_1 = d$, length = $2c$): Δ characterizes the conditions of loading:

$$\frac{S_g}{w_0} = \frac{1}{c_1} = \frac{2 + \frac{d}{2c}}{d : 2} = \frac{2 + \frac{d}{2c}}{d} = \frac{2}{d} + \frac{1}{c}.$$

The relative surface of the bomb diminishes as the diameter d and length $2c$ are increased. When volume w_0 is increased 8 times, S_g/w_0 diminishes approximately by one half.

Formula (17) shows that $\frac{\Delta p_m}{p_m}$ in a given bomb increases as Δ is reduced and hence the losses are particularly high at low values of Δ .

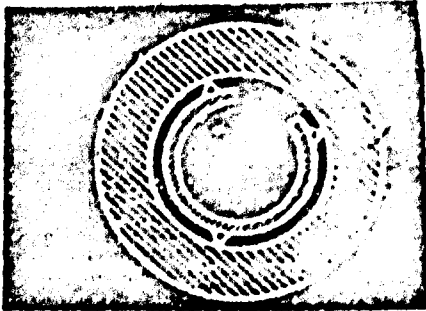


Fig. 17 - Test Arrangement for Calculating Losses due to Heat Transfer.

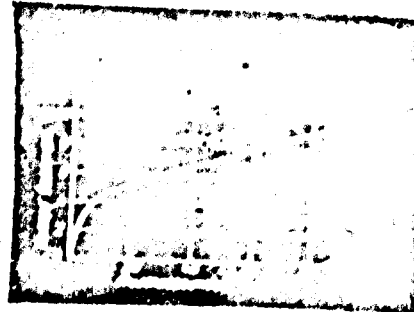


Fig. 18 - Curve C, Characterizing the Losses Due to Heat Transfer (according to Miuraur).

Ordinate: pressure loss in %;
abscissa: milliseconds; 1)
curve "C."

Large igniters are not necessary when working with conical crushers, because almost instantaneous ignition results at the low value of $p_{II} \approx 100-120 \text{ kg/cm}^2$.

Therefore, Miuraur's data cannot be directly applied to our test results and to powder tests for determining heat transfer, without conducting special tests. However, using the theoretical formula as a basis, it is possible to compute the burning time at $\Delta = 0.20$ and $p_B = 250 \text{ kg/cm}^2$ and to determine the order of the coefficient values C_M for powders of various thicknesses, in order to introduce the necessary correction for heat loss.

The table presented below contains the results of such calculations,

obtained from curve C.

t_k was determined from formula:

$$t_k = 2.303\tau \log \frac{p_m}{p_B},$$

where

$$\tau = \frac{c_1}{u_1} \frac{1 - \alpha \Delta}{f \Delta} = \frac{c_1}{u_1} \frac{1}{p_m}.$$

(for a powder with a constant burning surface area) (Table 8).

Table 8 - Values of coefficient C_M for pyroxylin powders

Thickness of powder $2e_1$ in mm	0.30	0.30 flegm.	0.40	1.00	2.00	4.00
Rate of combustion u_1 $\frac{dm}{sec} : \frac{kg}{dm^2} \cdot 10^7$	90	70	80	75	72	62
Coefficient $\frac{1}{\tau}$	1220	950	813	305	146	69
Burning time in milliseconds, at $\Delta = 0.20$ and $p_0 = 250 \text{ kg/cm}^2$	1.76	2.26	2.64	7.00	14.6	31.0
Heat transfer coefficient C_M in %	1.5	2.0	2.6	4.0	5.0	6.1

Corrections for heat loss during burning of powder in a bomb can be introduced by the aid of this table.

Let us introduce the correction for heat loss when determining f and α .

We shall assume that tests were conducted with a bomb having a volume $W_0 = 78.5 \text{ cm}^3$ at two loading densities $\Delta_1 = 0.15$ and $\Delta_2 = 0.25$. The powder is 1 mm thick.

$p_{m1} = 1435 \text{ kg/cm}^2$, $p_{m2} = 2760 \text{ kg/cm}^2$, $p_{m2} - p_{m1} = 1325$. We determine f and α without taking the heat-transfer losses into consideration:

$$\frac{p_{m1}}{\Delta_1} = 9570, \frac{p_{m2}}{\Delta_2} = 11,020; \frac{p_{m2}}{\Delta_2} - \frac{p_{m1}}{\Delta_1} = 1450.$$

$$\alpha = \frac{1450}{1325} = 1.096 \text{ dm}^3/\text{kg}$$

$$f = 957,000 - 1.096 \cdot 143,300 = 800,000 \text{ kg-dm/kg}.$$

We now introduce the correction for heat losses in pressures p_{m1} and p_{m2} , and determine the corrected values of energy f_0 and covolume α_0 .

$$\frac{\Delta p_m}{p_m} = \frac{S_g}{W_0} \frac{1}{\Delta} \frac{C_M\%}{7.774}.$$

According to the table, $C_M = 4\%$.

For a bomb of volume $W_0 = 78.5 \text{ cm}^3$

$$\frac{S_g}{W_0} = 1.30 \text{ cm}^2/\text{cm}^3.$$

$$\frac{S_g}{W_0} \frac{1}{7.774} = 0.1673; 0.1673 \cdot 4 = 0.6692.$$

$$\Delta p_m = \frac{S_g}{W_0} \frac{C_M\%}{7.774} \frac{p_m}{\Delta} \left\{ \begin{array}{l} \Delta p_{m1} = 0.6692 \cdot 9570 = 64 \text{ kg/cm}^2 \\ \Delta p_{m2} = 0.6692 \cdot 11,040 = 74 \text{ kg/cm}^2 \end{array} \right.$$

$$\begin{array}{l} p_{m1}' = p_{m1} + \Delta p_{m1} = 1435 + 64 = 1499 \text{ kg/cm}^2 \\ p_{m2}' = p_{m2} + \Delta p_{m2} = 2760 + 74 = 2834 \text{ kg/cm}^2 \end{array} \left| \begin{array}{l} \frac{p_{m1}'}{\Delta_1} = 10,000; \\ \frac{p_{m2}'}{\Delta_2} = 11,340 \end{array} \right.$$

$$p_{m2} - p_{m1} = 1335 \text{ kg/cm}^2;$$

$$\frac{p_{m2}}{\Delta_2} - \frac{p_{m1}}{\Delta_1} = 1340;$$

$$\alpha_0 = \frac{1340}{1335} = 1.004 \text{ dm}^3/\text{kg} \approx 1.00.$$

$$f_0 = 1,000,000 - 1.004 \cdot 1499 = 849,500 \text{ kg-dm/kg} \approx 850,000 \text{ kg-dm/kg}.$$

As can be seen from the above results, the introduction of a correction for the heat lost has increased the energy of the powder from $f = 800,000$ to $850,000 \text{ kg-dm/kg}$, i.e., by 6.25%, and reduced the magnitude of covolume from 1.096 to $1 \text{ dm}^3/\text{kg}$, i.e., by 9.6%.

The same powder burned in a bomb having a volume of 25.25 cm^3 , and at the same loading density, will give $p_{m1} = 1405 \text{ kg/cm}^2$ and $p_{m2} = 2725 \text{ kg/cm}^2$. On the basis of this data, the resultant magnitudes of energy f and covolume α will be:

$$f = 774,000 \text{ kg-dm/kg} \quad \text{and} \quad \alpha = 1.16 \text{ dm}^3/\text{kg}.$$

For this bomb

$$\frac{S_6}{w_0} = 1.89 \text{ cm}^2/\text{cm}^3.$$

Corrections in pressure for heat transfer will be as follows:

$$\Delta p_{m1} = 95 \text{ kg/cm}^2 \quad \text{and} \quad \Delta p_{m2} = 110 \text{ kg/cm}^2;$$

$$f = 850,000 \quad \text{and} \quad \alpha = 1.00.$$

In this way, magnitudes f and α for a bomb of small volume are obtained with a larger percentage of error: the energy will be

smaller and the covolume will be higher than their true values.

As shown by Kokhanov's experiments, these errors are extremely large for very low loading densities and thick powders.

The introduction of corrections for heat transfer losses by the above method provides practically identical values of f and α for all loading densities, and transforms the hyperbola $\frac{p_m}{\Delta}$, p_m obtained in Kokhanov's experiments into a straight line, as it should be [5].

The table presented below lists values of magnitudes $\frac{S_c}{W_0}$ and $\frac{S_c}{W_0} \cdot \frac{1}{7.774}$ for manometric bomb of the most typical dimensions (Table 9).

Table 9

$W_0, \text{ cm}^3$	21.8	25.25	78.5	120	146.5	216	Krupp's 3320 bomb
d_0	2.2	3.0	4.4	4.4	3.0	4.4	8.0
$\frac{S_c}{W_0}$	2.17	1.89	1.30	1.16	1.48	1.05	0.53
$\frac{S_c}{W_0} \cdot 7.774$	0.279	0.243	0.1673	0.1495	0.184	0.135	0.0682

Curve C_M is expressed as a function of the burning time t_k of powder at $\Delta = 0.20$.

Inasmuch as the burning time of powder t_k is directly proportional to the total pressure impulse $\int_0^1 p dt$, the dependence of C as a function of I_k can be plotted independently of the loading density. Work of this type was conducted by M.I. Samarina at the Naval Artillery Research Institute in 1938, at which time experiments were repeated with bombs with and without steel inserts. The pressure was recorded

by means of conical crushers. The values of $\frac{\Delta p_m}{p_m} \% = C_A \%$ were plotted as a function of l_k (fig. 19). The form of the curve differs somewhat from curve C_{Mlt_k} .

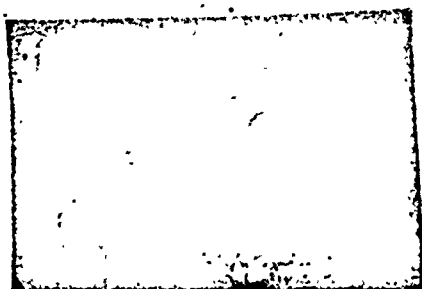


Fig. 19 - Curve C_A , Characterizing Losses due to Heat Transfer (according to the Naval Artillery Research Institute data).

Bomb tests for determining f and α involve heat losses in the form of heat transfer to walls of the bomb, and, therefore, pressures p_1 and p_2 contain an error in their determined values, δp_1 and δp_2 being inversely proportional to magnitudes $1 - \alpha \Delta_1$ and $1 - \alpha \Delta_2$, and $\delta p_2 > \delta p_1$; but $\frac{\delta p_1}{p_1} > \frac{\delta p_2}{p_2}$ because $\frac{\delta p}{p} \% = \frac{S_6}{w_0} \frac{1}{\Delta} \frac{C_M \%}{7.774}$, where C_M is the heat transfer coefficient.

Corrections for heat losses in the powder energy f and the magnitude of covolume can be determined after introducing corrections for heat losses in the values of pressures p_1 and p_2 , by adding the equalities (7), (8), (9) and (10), taking into consideration equality (a) (see Chapter II):

$$\delta f = \delta f_1 + \delta f_2 = \frac{\Delta_2 - \Delta_1}{\Delta_2 \cdot \Delta_1} \frac{1}{(p_2 - p_1)^2} (p_2^2 \delta p_1 - p_1^2 \delta p_2) =$$

$$= \frac{\Delta_2 - \Delta_1}{p_2 - p_1} \frac{p_2}{\Delta_2} \frac{p_1}{\Delta_1} \frac{p_2^2 \delta p_1 - p_1^2 \delta p_2}{(p_2 - p_1) p_2 p_1} = f \frac{p_2 \frac{\delta p_1}{p_1} - p_1 \frac{\delta p_2}{p_2}}{p_2 - p_1}.$$

Finally,

$$\frac{\delta f}{f} = \frac{p_2 \frac{\delta p_1}{p_1} - p_1 \frac{\delta p_2}{p_2}}{p_2 - p_1}; \quad (18)$$

$$\begin{aligned} \delta \alpha &= \delta \alpha_1 + \delta \alpha_2 = -\frac{\frac{1}{\Delta_1} - \frac{1}{\Delta_2}}{(p_2 - p_1)^2} (p_2 \delta p_1 - p_1 \delta p_2) = \\ &= -f \frac{p_2 \delta p_1 - p_1 \delta p_2}{(p_2 - p_1) p_1 p_2} = -f \frac{\frac{\delta p_1}{p_1} - \frac{\delta p_2}{p_2}}{p_2 - p_1}. \end{aligned} \quad (19)$$

When the corrections for heat transfer are taken into account

$$\frac{\delta p}{p} \% = \frac{C_M \%}{7.774} \frac{S_g}{W_0} \frac{1}{\Delta}.$$

For a given powder, $C_M = \text{const}$, and for a given bomb, $\frac{S_g}{W_0} = \text{const}$, only Δ changes.

Designating

$$\frac{C_M \%}{7.774} \frac{S_g}{W_0} = D,$$

we get

$$\frac{\delta p_1}{p_1} = \frac{D}{\Delta_1}; \quad \frac{\delta p_2}{p_2} = \frac{D}{\Delta_2}.$$

Substituting these expressions in formulas (18) and (19), we obtain:

$$\frac{\delta f}{f} \% = \frac{D \left(\frac{p_2}{\Delta_1} - \frac{p_1}{\Delta_2} \right)}{p_2 - p_1}; \quad (20)$$

$$\delta \alpha = -fD \frac{\frac{1}{\Delta_1} - \frac{1}{\Delta_2}}{p_2 - p_1} = -\frac{f^2 D}{p_1 \cdot p_2}. \quad (21)$$

Let us apply the example presented above.

Example. 1 mm thick powder is burned in a bomb having a volume of 78.5 cm³; the powder coefficient is $C_M = 4\%$; and the value of $\frac{S_f}{W_0}$ for this bomb = 1.30 cm²/cm³.

$$\frac{S_f}{W_0} \frac{C_M\%}{7.774} = 0.1673 \cdot 4 = 0.6692\% = 0.006692 = D.$$

$$\begin{array}{l} \Delta_1 = 0.15; p_1 = 1433; \frac{p_1}{\Delta_1} = 9570 \\ \Delta_2 = 0.25; p_2 = 2756; \frac{p_2}{\Delta_2} = 11,020 \end{array} \left| \begin{array}{l} \text{Without correction for heat} \\ \text{transfer losses} \\ f = 800,000 \text{ kg-dm/kg;} \\ \alpha = 1.096 \text{ dm}^3/\text{kg.} \end{array} \right.$$

We shall now calculate the corrections for f and α taking the heat-transfer into account:

$$\frac{\delta f\%}{f} = 0.6692 \frac{\frac{2756}{0.15} - \frac{1433}{0.25}}{2756 - 1433} = 0.6692 \frac{18,350 - 5730}{1323} = 6.4\%;$$

$$f_0 = f \left(1 + \frac{\delta f}{f}\right) = 800,000 \cdot 1.064 = 852,000 \text{ kg-dm/kg};$$

$$\delta \alpha = -\frac{f^2 \cdot D}{p_1 p_2} = -\frac{800,000^2 \cdot 0.006692}{143,300 \cdot 275,600} = -0.1085;$$

$$\alpha_0 = \alpha + \delta \alpha = 1.096 - 0.1085 = 0.9875 \approx 0.99.$$

CHAPTER IV - THE LAW OF GAS FORMATION

1. DEFINITION

The study of the law of gas formation under the simplest conditions, in a constant volume, permits the application of the obtained relationships to the determination of pressure in the bore of a weapon when the gun is fired, i.e., under conditions of variable volume.

The general pyrostatics formula

$$p_{\psi} = \frac{f\omega\psi}{W_0 - \frac{\omega}{\delta} - \left(\alpha - \frac{1}{\delta}\right)\omega\psi} = \frac{f\omega\psi}{W_{\psi}}$$

shows that the magnitude of gas pressure at given conditions of loading ($W_0, \omega, f, \alpha, \delta$) is governed by the amount of the burned portion of the charge ψ , where $\omega\psi$ is the gravimetric inflow of powder gases at a given instant, and $f\omega\psi$ is the inflow of the energy contained in this quantity of gas.

Keeping in mind that W_{ψ} varies slightly under conditions prevailing in a manometric bomb, it may be said that the pressure is almost proportional to the burned portion of the charge ψ at the given powder energy f and the given loading density.

Correspondingly, the nature of pressure increase in time $\frac{dp}{dt}$ under the same conditions of loading is also determined in the main by the change of magnitude $\frac{d\psi}{dt}$ with time.

The law of gas formation is a term defining the change of the magnitude of ψ with time and of its derivative $\frac{d\psi}{dt}$, known as the "rate of gas formation" or "volumetric rate of burning."

An analysis of this magnitude $\frac{d\psi}{dt}$ makes it possible to determine the means by which the gas inflow during burning of powder can be regulated.

2. RATE OF GAS FORMATION

We shall derive the formula for the rate of gas formation, based on the burning of powder in parallel layers.

Let us assume that the burning of a powder grain of arbitrary shape proceeds in concentric layers at a constant rate in all directions. At some instant the grain, whose initial volume is Λ_1 and whose initial surface is S_1 , has a volume Λ and a surface S (fig. 1). We shall also assume that a layer of thickness de is burned during the time interval dt . Then, the volume burned during the time interval dt will be expressed by formula:

$$d\Lambda_{cr} = Sde,$$

whence

$$\Lambda_{cr} = \int_0^e Sde;$$

$$\psi = \frac{\Lambda_{cr}}{\Lambda_1} = \frac{\int_0^e Sde}{\Lambda_1}.$$

Differentiating both sides of this equality with respect to t , we get:

$$\frac{d\psi}{dt} = \frac{\frac{\Lambda_{cr}}{\Lambda_1}}{\frac{dt}{dt}} = \frac{S}{\Lambda_1} \frac{de}{dt} = \frac{S}{\Lambda_1} u.$$

Multiplying and dividing the right side by S_1 , we get:

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1} \frac{S}{S_1} u.$$

The first two multiples in the right side of the expression depend on the geometry of the powder grain:

$\frac{S_1}{\Lambda_1}$ - the initial exposed area of the powder grain or the specific surface per unit grain volume at the start of burning; it will be shown later that this area depends on the form and the dimensions of the grain;

$\frac{S}{S_1}$ - the relative surface of a powder grain; it varies during burning and depends only on the form of the grain and on the relative thickness of the burnt powder layer, but not on its absolute dimensions.

As will be shown later, the third multiple, the linear rate of burning $u = \frac{de}{dt}$, depends on the type of powder, the pressure under which the powder burns, and on its temperature.

In order to determine the rate of burning in an actual test bomb under variable pressure, it is necessary to know the thickness variation of the burning layer per unit of time, and to this end it is necessary to establish the purely geometrical relation between the thickness of the burned layer and the volume of the powder gases in their various forms.

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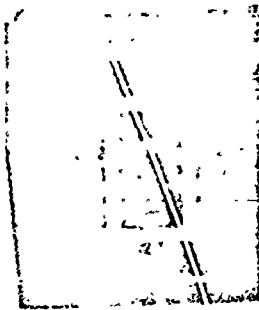


Fig. 20 - Diagram of Powder Burning in Parallel Layers.

If the pressure curve plotted as a function of time, or a table of the values of p versus t is available from the bomb test, the values of ψ , t can be calculated by means of the general pyrostatics formula (or from special tables), following which the geometrical law of burning can be applied to establish the relationship between the thickness of the powder, the value of ψ and $\frac{S}{S_1}$, and to derive the dependence of the initial exposure $\frac{S_1}{\Lambda_1}$ on the form and dimensions of the grains. The numerical differentiation of ψ and e with respect to time t will permit obtaining from the experiment the rate of gas formation and the linear rate of burning, as well as their variation, during the burning of the powder.

The establishment of all these relationships will facilitate the analysis of those factors which can be used to control the quantity and the intensity of gas formation during the burning of powder and therefore, will permit the control of the phenomena of a gun discharge.

3. THE EFFECT PRODUCED BY THE GEOMETRY OF A POWDER GRAIN ON GAS FORMATION

The inflow of gases per unit of time for a powder of a given type (f, α, δ, u_1) can be regulated by the loading density or by the dimensions and the shape of the powder grains.

The effect of the geometry of the given grains on the rate of gas formation depending on the thickness of the powder burned at the given instant, can be determined by means of the basic conditions of the geometrical law of combustion.

The geometrical law of combustion permits determining the relationship between the relative thickness of the powder burned at the given instant $z = \frac{e}{e_1}$, the burnt portion of the grain $\psi = \frac{\Lambda_{cr}}{\Lambda_1}$ (Λ_{cr} being the volume of the burnt portion of the powder), and the relative surface of the powder $\sigma = S/S_1$ at the same instant. The dependence of the product $\Sigma = \frac{S_1}{\Lambda_1} \frac{S}{S_1}$ on the shape and the dimensions of the powder (grain) can be established at the same time, which product enters the formula for determining the rate of gas formation and has a great effect on the law governing the gas pressure development during a discharge.

A. Relation Between the Burnt Portion of Powder ψ and the Relative Thickness of Powder z , Consumed at the Same Instant (Inflow of Gases)

Investigations show that the dependence of ψ on z for all forms of powder is expressed by a formula of the same type(*):

(*) In deriving this formula for one grain, as well as for an entire charge composed of many grains, it is assumed that all grains of the charge have strictly identical dimensions and are regular in form, the latter being bounded by parallel planes, intersecting at right angles or by surfaces of revolution whose axes coincide or are parallel to each other.

$$\psi = \chi z (1 + \lambda z + \mu z^2), \quad (22)$$

where χ , λ , μ are shape characteristics - constant values depending on the shape of the grain; they possess a particular numerical value for each grain shape inherent to the given grain form.

The thickness of the burnt layer e varies during burning from 0 to e_1 ; the relative thickness z varies from 0 to 1; and the relative volume ψ fluctuates between 0 and 1.

We shall now derive the dependence of ψ on z for strip powder (a parallelepiped with three different dimensions) (fig. 21). We shall introduce the designations: $2e_1$ for the thickness of the strip, $2b$ for the width of the strip, and $2c$ for its length:

$$\frac{2e_1}{2b} = \alpha; \quad \frac{2e_1}{2c} = \beta.$$

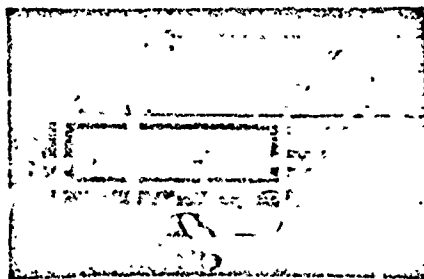


Fig. 21 - Burning Diagram of a Powder Strip.

Magnitudes α and β characterize the span of the strip in thickness and length. Furthermore, inasmuch as all the dimensions become reduced in all directions by a magnitude $2e_1$ during the full burning of the powder, $\alpha = \frac{2e_1}{2b}$ represents the relative reduction of the strip in width and $\beta = \frac{2e_1}{2c}$ represents the relative reduction of its length during the full burning period of the powder.

Inasmuch as

$$2e_1 < 2b < 2c,$$

$$1 > \alpha > \beta > 0.$$

Let us assume that at the given instant a layer of powder of thickness e will be burned on all sides. The volume burned can be determined more easily as the difference between the initial volume Λ_1 and the remaining volume Λ_{ocr} .

We will have (see fig. 21):

$$\psi = \frac{\Lambda_{ocr}}{\Lambda_1} = \frac{\Lambda_1 - \Lambda_{ocr}}{\Lambda_1} = 1 - \frac{\Lambda_{ocr}}{\Lambda_1};$$

$$\Lambda_1 = 2e_1 \cdot 2b \cdot 2c;$$

$$\Lambda_{ocr} = (2e_1 - 2e) (2b - 2e) (2c - 2e);$$

$$\frac{\Lambda_{ocr}}{\Lambda_1} = \frac{e_1 - e}{e_1} \frac{b - e}{b} \frac{c - e}{c} = \left(1 - \frac{e}{e_1}\right) \left(1 - \frac{e}{b}\right) \left(1 - \frac{e}{c}\right);$$

but

$$\frac{e}{e_1} = z, \quad \frac{e}{b} = \frac{e_1}{b} \frac{e}{e_1} = \alpha z; \quad \frac{e}{c} = \frac{e_1}{c} \frac{e}{e_1} = \beta z;$$

then

$$\frac{\Lambda_{ocr}}{\Lambda_1} = (1 - z) (1 - \alpha z) (1 - \beta z).$$

Upon removing the parentheses we obtain:

$$\frac{\Lambda_{ocr}}{\Lambda_1} = 1 - (1 + \alpha + \beta)z + (\alpha + \beta + \alpha\beta)z^2 - \alpha\beta z^3.$$

Substituting this expression in the formula for ψ , we find:

$$\psi = (1 + \alpha + \beta)z - (\alpha + \beta + \alpha\beta)z^2 + \alpha\beta z^3,$$

or, reducing it to the general form of equation (22), we get:

$$\psi = (1 + \alpha + \beta)z \left[1 - \frac{\alpha + \beta + \alpha\beta}{1 + \alpha + \beta}z + \frac{\alpha\beta}{1 + \alpha + \beta}z^2 \right].$$

Introducing the designations

$$1 + \alpha + \beta = \kappa; \quad -\frac{\alpha + \beta + \alpha\beta}{1 + \alpha + \beta} = \lambda; \quad \frac{\alpha\beta}{1 + \alpha + \beta} = \mu, \quad (23)$$

we obtain a general type formula:

$$\psi = \kappa z (1 + \lambda z + \mu z^2).$$

At the end of burning at $z = 1$ $\psi = 1$, and formula (23) assumes an equality in the form

$$1 = \kappa (1 + \lambda + \mu), \quad (24)$$

which must be satisfied by the numerical characteristics κ , λ and μ .

This equality serves to verify the values of the characteristics calculated by means of formula (23).

The characteristics κ , λ and μ depend on the ratio of the dimensions: the shorter and the narrower the strip, the greater is α and β ; the longer the strip, the closer is κ to unity and λ and μ to zero.

3. The Law Governing the Change of the Powder Surface When the Powder is Burned.

Formula (22) is a general formula for all powder shapes; the difference will be only in the numerical values of the characteristics κ , λ and μ . Using this formula as a basis, we shall derive a formula for depicting the relative surface $\sigma = \frac{S}{S_1}$ and the initial exposure $\frac{S_1}{\Lambda_1}$, characterizing the effect of the shape and dimensions of the powder on the rate of gas formation.

Differentiating ψ with respect to z , we find:

$$\frac{d\psi}{dz} = \kappa(1 + 2\lambda z + 3\mu z^2). \quad (25)$$

Inasmuch as

$$\frac{d\psi}{dz} = \frac{d\psi}{dt} \frac{dt}{de} \frac{de}{dz},$$

and

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1} \cdot \frac{S}{S_1} u, \quad \frac{dt}{de} = \frac{1}{u}, \quad \frac{de}{dz} = \frac{de}{d\frac{e}{e_1}} = e_1,$$

then

$$\frac{d\psi}{dz} = \frac{S_1}{\Lambda_1} \cdot \frac{S}{S_1} u \frac{e_1}{u} = \frac{S_1}{\Lambda_1} e_1 \frac{S}{S_1}.$$

Substituting this expression in the left side of equation (25), we obtain

$$\frac{S_1}{\Lambda_1} e_1 \frac{S}{S_1} = \kappa (1 + 2\lambda z + 3\mu z^2). \quad (26)$$

At the start of burning $z = 0$, $S = S_1$, $\frac{S}{S_1} = 1$, and equation (26) at the start of burning will be written as follows:

$$\frac{S_1}{\Lambda_1} e_1 = \kappa. \quad (27)$$

Dividing each term of (26) by (27), we will find the desired relationship:

$$\epsilon = \frac{S}{S_1} = 1 + 2\lambda z + 3\mu z^2. \quad (28)$$

At the start of burning, at $z = 0$,

$$\epsilon = 1;$$

at the end of burning, at $z = 1$,

$$\epsilon_K = \frac{S_K}{S_1} = 1 + 2\lambda + 3\mu.$$

While the powder burns, and z varies from 0 to 1, the change of δ will mainly depend on the magnitude and sign of the characteristic λ , since μ is small compared to λ .

If the surface of the powder diminishes while burning (strip, cube, bar, $\lambda < 0$) the shape of such a powder is called regressive; if the surface area becomes greater during burning (powder with multiple perforations, $\lambda > 0$), the powder is called progressive.

The δ function depends on λ and μ , i.e., on the shape of the grain and its dimensional ratio, rather than on the absolute dimensions of the powder. The greater the value of $|\lambda|$, the greater will be the surface change of the powder in burning.

From equality (27) we obtain an expression for the initial exposure which is of great importance in controlling the rate of gas formation:

$$\frac{S_1}{\Lambda_1} = \frac{\kappa}{e_1}; \quad (29)$$

this equality shows that the initial burning area of the powder depends on its shape (characteristic κ), as well as on its dimensions (e_1).

The smaller the value of e_1 , the thinner is the powder, and the greater will be the quantity of gases which it will evolve per unit of time.

Substituting (28) in (25), we obtain formula

$$\frac{d\psi}{dz} = \kappa \cdot \delta,$$

which will be used in plotting the graph for ψ , z .

C. Determination of ψ , z for Other Grain Shapes.

a) Tubular grain with a single perforation (fig. 22).

Designating:

$2e_1$ - web thickness

D - outside diameter

d - inside diameter

$2c$ - length of tube;

$$\frac{2e_1}{2c} = \frac{e_1}{c} = \beta \text{ (which is the same as for strip powder);}$$

$$\Lambda_1 = \frac{\pi}{4} (D^2 - d^2) 2c;$$

$$\Lambda_{\text{ост}} = \frac{\pi}{4} [(D - 2e)^2 - (d + 2e)^2] (2c - 2e);$$

$$\frac{\Lambda_{\text{ост}}}{\Lambda_1} = \frac{[(D - 2e)^2 - (d + 2e)^2]}{D^2 - d^2} \frac{(c - e)}{c}.$$

GRAPHIC NOT REPRODUCIBLE



Fig. 22 - Burning of a Tubular Grain.

Removing the parentheses and bearing in mind that

$$D - d = 2 \cdot 2e_1,$$

and

$$\frac{e}{c} = \frac{e_1}{c} \frac{e}{e_1} = \beta \cdot z,$$

we get:

$$\begin{aligned} \frac{\Lambda_{\text{oct}}}{\Lambda_1} &= \frac{\sqrt{D^2 - d^2 - 2(D + d)2e}}{D^2 - d^2} \left(1 - \frac{e}{c}\right) = \frac{(D - d - 2 \cdot 2e)}{D - d} \left(1 - \frac{e}{c}\right) \\ &= (1 - z) (1 - \beta z) = 1 - (1 + \beta)z + \beta z^2; \end{aligned}$$

$$\psi = 1 - \frac{\Lambda_{\text{oct}}}{\Lambda_1} = (1 + \beta)z - \beta z^2 = (1 + \beta)z \left(1 - \frac{\beta}{1 + \beta}z\right).$$

Assuming $1 + \beta = \kappa$, $-\frac{\beta}{1 + \beta} = \lambda$ and $\mu = 0$, we get once again a general type formula

$$\psi = \kappa z (1 + \lambda z),$$

where the terms μz^2 is missing, because $\mu = 0$.

It can be easily seen that a tube is equivalent to a strip whose dimension in width does not change in burning. This is equivalent to a strip of infinite width, where $\alpha = 0$.

In such a case, the characteristics of strip powder will be

$$\kappa = 1 + 0 + \beta = 1 + \beta;$$

$$\lambda = - \frac{0 + \beta + 0}{1 + \beta} = - \frac{\beta}{1 + \beta};$$

$$\mu = 0,$$

i.e., the same as those obtained for tubular powder.

It is of interest to note that characteristics κ and λ for a tubular grain do not depend on the diameter, but, rather, on the web thickness $2e_1$ and the length of the tube $2c$.

b) Grain shapes which are derivatives of strip powder

It can be easily seen that the following grain shapes can be obtained from a strip as special cases of the latter:

- 1) square rod: $2b = 2c; \alpha = \beta;$
- 2) square slab: $2e_1 = 2b; \alpha = 1; \beta > 0;$
- 3) cube: $2e_1 = 2b = 2c; \alpha = \beta = 1.$

The characteristics of these shapes, as well as of strips and tubes, are given below in Table 10; also given in this table are the values of G_K at the end of powder burning.

Table 10

Powder Shape	λ	λ	μ	$\sigma_K = 1 + 2\lambda + 3\mu$
Tube (infinitely wide strip)	$1 + \beta$	$-\frac{\beta}{1 + \beta}$	0	$\frac{1 - \beta}{1 + \beta}$
Strip	$1 + \alpha + \beta$	$-\frac{\alpha + \beta + \alpha\beta}{1 + \alpha + \beta}$	$\frac{\alpha\beta}{1 + \alpha + \beta}$	$\frac{(1 - \alpha)(1 - \beta)}{1 + \alpha + \beta}$
Square Rod	$1 + 2\beta$	$-\frac{2\beta + \beta^2}{1 + 2\beta}$	$\frac{\beta^2}{1 + 2\beta}$	$\frac{(1 - \beta)^2}{1 + 2\beta}$
Square Slab	$2 + \beta$	$-\frac{1 + 2\beta}{2 + \beta}$	$\frac{\beta}{2 + \beta}$	0
Cube	3	-1	$\frac{1}{3}$	0

The data presented in the table shows that all the grain characteristics are increased in changing over from a tubular to a cube shape: λ increases from 1 to 3, $|\lambda|$ - from a small fraction to 1, μ - from 0 to 1/3.

Since λ characterizes the initial surface area $\frac{S_1}{\Lambda_1}$ for a given powder thickness $2e_1$, the increase of λ shows that in changing over from a strip to a cube, with the web thickness $2e_1$ remaining the same, the initial surface area is increased almost three times whereas the simultaneous increase of λ indicates a more drastic reduction of the surface area.

The diagram in fig. 23 clarifies the above: the heavy broken line divides the strip into square rods, the dotted line divides it into square slabs, while the dot-and-dash line divides it into cubes of the same thickness as that of the strip.

Increasing the tube length will ultimately result in a powder grain with a constant burning surface, for which

$$\kappa = 1, \lambda = 0, \mu = 0;$$

consequently, the relations between ψ, z and ζ, z will be expressed by the following formulas:

$$\psi = z; \zeta = 1.$$

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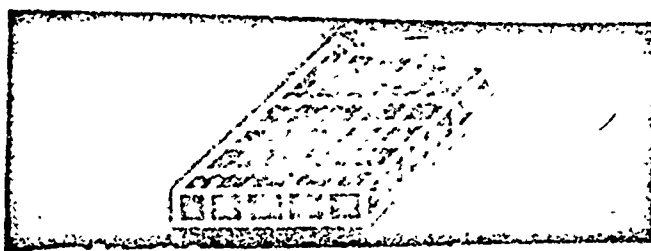


Fig. 23 - A Strip Divided into Derivative Shapes.

The same law governing the burning of powder will apply to tubular powder inhibited at its end..

Examples of Calculating the Characteristics of Powder Shapes

1. Strip powder CП (SP); dimensions in millimeters: 1 by 18 by 300.

$$\alpha = \frac{2e_1}{2b} = \frac{1}{18} = 0.05555; \kappa = 1 + \alpha + \beta = 1.0589;$$

$$\beta = \frac{2e_1}{2c} = \frac{1}{300} = 0.00333; \lambda = \frac{-(\alpha + \beta + \alpha\beta)}{\kappa} = \frac{-0.05907}{1.0589} = -0.05575;$$

$$\alpha\beta = 0.00019; \mu = \frac{\alpha\beta}{\kappa} = \frac{0.00019}{1.0589} = 0.00018;$$

$$\zeta_K = 1 + 2\lambda + 3\mu = 1 - 0.11150 + 0.00054 \approx 0.889;$$

$$\frac{S_1}{\Lambda_1} = \frac{\kappa}{e_1} = \frac{1.0589}{0.50} = 2.1178 \text{ mm}^2/\text{mm}^2.$$

For the same powder reduced to 40 mm in length, the characteristics will change as follows:

$$\kappa = 1.086; \lambda = -0.0758; \mu = 0.00129; \epsilon_K \approx 0.852; \frac{S_1}{\Lambda_1} = 2.161.$$

2. A tubular grain having the same wall thickness and length as the strip powder:

$$2e_1 = 1; 2c = 300;$$

$$\alpha = 0, \beta = \frac{1}{300} = 0.00333; \alpha\beta = 0.$$

$$\kappa = 1 + \beta = 1.00333; \lambda = -\frac{\beta_1}{1 + \beta} = -\frac{0.00333}{1.0033} = -0.00332;$$

$$\epsilon_K = \frac{S_K}{S_1} = 1 + 2\lambda = 1 - 0.00664 = 0.9934;$$

$$\frac{S_1}{\Lambda_1} = \frac{\kappa}{e_1} = \frac{1.0033}{0.5} = 2.0066.$$

The same tube 40 mm in length:

$$\kappa = 1.025; \lambda = -0.0244;$$

$$\frac{S_1}{\Lambda_1} = 2.050; \epsilon_K = 0.9512.$$

Compared to a strip of the same length, a tube has a smaller effective surface area and a lower degree of regression. The surface area and regression in the burning area increase with decrease in length.

If we calculate the characteristics for shapes having the form of solids of revolution, we will find that their characteristics are expressed by precisely the same ratios as given in the table of characteristics for parallelepipeds; the only changes occur in the values of α and β , which are expressed in terms of the dimensions of the solids of revolution.

Table 11, below, contains shape characteristics for solids of revolution in the form of a solid rod, sphere and tube.

Table 11 - Shape Characteristics of Solids of Revolution

Basic dimensions			$\alpha = \frac{e_1}{b}$	$\beta = \frac{e_1}{c}$	$\alpha\beta$	$\kappa = 1 + \alpha + \beta$	$\lambda = -\frac{\alpha + \beta + \alpha\beta}{1 + \alpha + \beta}$	$\mu = \frac{\alpha\beta}{1 + \alpha + \beta}$
$2e_1$	$2b$	$2c$						
Sphere								
$2R$	$2R$	$2R$	1	1	1	3	$-\frac{3}{3} = -1$	$\frac{1}{3}$
Solid cylinder (rod)								
$2R$	$2R$	$2c$	1	$\frac{R}{L_0} = \beta$	β	$2 + \beta$	$-\frac{1 + 2\beta}{2 + \beta}$	$\frac{\beta}{2 + \beta}$
Solid cylinder whose diameter equals its height								
$2R$	$2R$	$2R$	1	1	1	3	-1	$\frac{1}{3}$
Cylindrical plate (pellet)								
$2e_1$	$2R$	$2R$	$\frac{e_1}{R} = \beta$	$\frac{e_1}{R} = \beta$	β^2	$1 + 2\beta$	$-\frac{2\beta + \beta^2}{1 + 2\beta}$	$\frac{\beta^2}{1 + 2\beta}$
Tube								
$R-r$	∞	$2c$	0	$\frac{R-r}{2c} = \beta$	0	$1 + \beta$	$-\frac{\beta}{1 + \beta}$	0

Table 11 (Cont'd.)

Ring-shaped pellet

$\frac{2c}{(2c_0)}$	∞	$R-r$	0	$\frac{2e_1}{R-r} = \beta$	0	$1 + \beta$	$-\frac{\beta}{1 + \beta}$	0
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A comparison of the data in Tables 10 and 11 will show that the characteristics κ , λ , μ of some of the grain shapes listed in these tables are identical, and that therefore the law governing the change of volume ψ and surface area G as a function of the relative burned thickness z is the same. Such shapes are termed equivalent shapes.

They are exemplified, for example, by a cube, a sphere, a cylinder whose height equals its diameter, or by square and round slabs, and square and round bars.

D. Graphical Illustration of the Relationships Between
 $G - z$, $\psi - z$, $G - \psi$

Knowing the general expressions for the inflow of gases

$$\psi = \kappa z(1 + \lambda z + \mu z^2)$$

and for the law governing the change of relative surface

$$G = 1 + 2\lambda z + 3\mu z^2,$$

we can by assigning specific values to z , calculate the corresponding values of G and ψ and plot a graph for the investigated regressive grain shapes.

These diagrams are termed "progressive data sheets."

We shall consider the following shapes: 1 - tube; 2 - strip; 3 - square plate; 4 - a solid slab; 5 - cube or sphere.

a) ϵ, z diagram (fig. 24) (progressive data sheet).

Since λ is negative and μ is small in all the shapes under consideration, curves ϵ, z will always lie below the horizontal dotted line 1-1, which corresponds to a powder with a constant burning surface area.

At the start of burning, when $z = 0$, $\epsilon = 1$ for all powder shapes. At the end of burning, when $z = 1$, the values of ϵ_K will vary for different grain shapes. The expressions for these values are given in Table 10. Inasmuch as for a tubular grain $\mu = 0$, the expression $\epsilon = 1 + 2\lambda z$ will depict a straight line with an angular coefficient 2λ , where $\lambda < 0$; the straight line 1 is very close to the horizontal 1-1.

For strip powder, as well as all the other powder shapes, $\epsilon = 1 + 2\lambda z + 3\mu z^2$ where $\lambda < 0$.

$$\frac{d\epsilon}{dz} = 2\lambda + 6\mu z; \quad \frac{d^2\epsilon}{dz^2} = 6\mu > 0.$$

Consequently, curve ϵ, z is convex with respect to the z -axis. At the start of burning

$$\left(\frac{d\epsilon}{dz}\right)_0 = 2\lambda < 0.$$

Since the value of μ is small for a strip, the curvature of the line is very slight; the values of λ and μ increase as the shape

becomes more regressive, the angle of inclination of the tangent at the origin of the coordinate axes increases, as does the curvature of the curve itself. In the case of a solid slab the curve has a pronounced downward bulge, and passes through zero ($G_K = 0$) when $z = 1$ (curve 4).

In the case of a cube the downward slope and the convexity are maximum, when $G_K = 0$, $z = 1$ (curve 5).

Thus the diagram shows that the cube is the most regressive of the five powder shapes considered here; the surface of the cube diminishes abruptly at the very start of burning and approaches zero at the end of burning. The least regressive powder shape is the tube, whose burning (surface) area remains practically constant at all times (reduction does not exceed 1%). Powder in the shape of a cube, at a given thickness of elements, gives off a maximum quantity of gases per unit of time at the start of burning; this quantity diminishes rapidly with burning. On the other hand, the quantity of gases given off by a tubular powder grain remains practically constant.

b) ψ , z Diagram (fig. 25):

$$\psi = \kappa z(1 + \lambda z + \mu z^2).$$

Previously, we had the expression (9):

$$\frac{d\psi}{dz} = \kappa(1 + 2\lambda z + 3\mu z^2) = \kappa G;$$

when $z = 0$ $G = 1$ and $\left(\frac{d\psi}{dz}\right)_0 = \kappa.$

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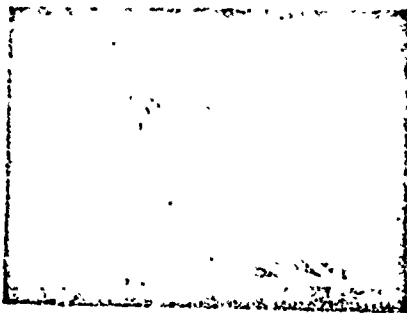


Fig. 24 - Change in area during regressive burning of powders of different shapes $\sigma = f(z)$.

Fig. 25 - The effect of grain shape on gas inflow for regressive powders $\psi = F(z)$.

$\frac{d\psi}{dz}$ is the tangent of the slope angle of curve ψ, z with the abscissa, while the shape characteristic κ represents the slope angle of the curve at the origin of the coordinates.

When $z = 0$ $\psi = 0$, $\sigma = 1$; when $z = 1$ $\psi = 1$.

All the curves are located within a square whose sides equal unity.

All the curves originate at the origin of the coordinates and pass through point $\psi = 1, z = 1$; the tangent of the angle of inclination at the origin is $\left(\frac{d\psi}{dz}\right) = \kappa$. Further variation of the angle of inclination is characterized by the value of σ . Inasmuch as σ diminishes in all the powder shapes considered here, the angle of inclination of all the curves diminishes also, and hence all the curves are convex upwards. In the case of a tubular grain κ approaches unity and the change of σ is small; curve 1 practically merges with a diagonal drawn from the origin of the coordinate system. For a cube, $\kappa = 3$ is maximum; curve 5 has the greatest angle of inclination at the origin and the greatest variation of this angle corresponds to the variation of σ . In the case of a cube and a slab $\sigma_K = 0$,

and hence the curves are tangent to the horizontal line 1-1 when $z = 1$.

The arrangement of the remaining curves 2, 3, and 4 is obvious and does not require any explanation.

The diagram shows that for a given value of z the portion of the burnt grain ψ will be the larger, the more regressive is the powder and the greater is χ . For example, in the case of tubular grain, at the instant the first half of the thickness ($z = 0.5$) is burned, the burnt portion of volume ψ will be equal to 0.5005, and in the case of a cube, when $z = 0.5$, $\psi = \frac{7}{8} = 0.875$.

Consequently, a more regressive powder gives off a larger quantity of gases during the first half of the burning process, and a smaller such quantity, during the second half.

c) The ϕ, ψ diagram has the greatest practical value, because it can be more easily compared with experimental data when evaluating the pressure curves obtained in the burning of powder in a manometric bomb. ψ is determined from the value of p by the aid of the general formula of pyrostatics, while ϕ goes into $\frac{d\psi}{dt}$ obtained by the numerical differentiation of the dependence of ψ on t . When this data is available, a comparison can be made of the theoretical and the experimental results.

Inasmuch as the equation for ψ , z is a third power equation, and ϕ , z is a second power equation, z is usually not eliminated when determining the dependence of ϕ, ψ ; rather, by assigning definite values to z , the corresponding values of ψ and ϕ are computed, and the results are then plotted on the ϕ, ψ diagram.

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How will the ϕ , z diagram change after it is transformed into a ϕ , ψ diagram (fig. 26)?

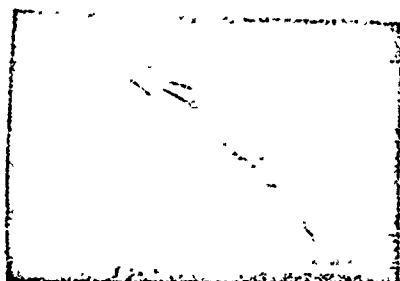


Fig. 26 - Effect of the Shape of Regressive Powder Grains on the Change of the Powder Area During Burning - $\phi = \psi$.

Similarly to the ϕ , z diagram, $\phi = 1$ when $\psi = 0$. At the end of burning when $\psi = 1$, ϕ_K will have the same values as when $z = 1$. Consequently, the position of the initial and final points will not change.

Since all curves of ψ , z are situated above the diagonal dividing the square ψ , z in half, the magnitude $\psi > z$ will correspond to some value of z , and this magnitude will be the greater, the more regressive is curve ψ , z .

Therefore, when ψ is replaced by z , all the points on curves ϕ , z (see fig. 24), while remaining at the same height, will shift to the right and the amount of displacement will be the greater, the more regressive is curve ψ , z or ϕ , z (see fig. 26).

d) A binomial formula for the relationship ψ , z .

The examples given here for calculating the characteristics κ , λ , μ for strip type powder show that μ is very small for strip and plate type grains, and that the term μz^2 does not appreciably affect the law governing the variation of ψ and ϕ . Therefore, in order to

simplify the expression subsequently entering the rather complex formulas of pyrodynamics in the solution of the basic problem, a binomial formula is used to express ψ for strip type powders without impairing the accuracy, by neglecting the term μz^2 in parenthesis. The influence of the neglected term μz^2 is compensated for by changing the remaining characteristics κ and λ , based on the following considerations.

Having a complete trinomial formula

$$\psi = \kappa z(1 + \lambda z + \mu z^2)$$

with known characteristics, we shall replace it by a binomial formula with new characteristics κ_1 and λ_1 :

$$\psi = \kappa_1 z(1 + \lambda_1 z).$$

In both equations $\psi = 0$ when $z = 0$. We shall establish the following conditions in order to determine coefficients κ_1 and λ_1 : 1) when $z = 1$ (end of burning), the binomial formula must give us $\psi = 1$, and 2) when $z = 0.5$, the value of ψ determined by means of the binomial formula must have the same value as ψ found by means of the trinomial formula at the same value of $z = 0.5$.

We thus obtain a system of two equations with two unknown coefficients κ_1 and λ_1 :

when $z = 1$

$$\kappa(1 + \lambda + \mu) = 1 = \kappa_1(1 + \lambda_1);$$

when $z = 0.5$

$$\frac{x}{2} \left(1 + \frac{\lambda}{2} + \frac{\mu}{4} \right) = \frac{x_1}{2} \left(1 + \frac{\lambda_1}{2} \right).$$

Solving this system, we get [12]:

$$x_1 = x - \frac{x\mu}{2} = x \left(1 - \frac{\mu}{2} \right).$$

We can obtain from the first equation of the system

$$\lambda_1 = \frac{1}{x_1} - 1;$$

$$\lambda_1 = \frac{\lambda + \frac{3}{2}\mu}{1 - \frac{\mu}{2}}.$$

Since both λ_1 and $\lambda < 0$, for the absolute values thereof

$$|\lambda_1| = \frac{|\lambda| - \frac{3}{2}\mu}{1 - \frac{\mu}{2}}.$$

When the values of the characteristics x_1 and λ_1 are determined in this manner, the second degree curve ψ, z and the third degree curve ψ, z will coincide at the starting point $z = 0$, then at $z = 0.5$, and finally at the end point $z = 1$.

Thus, in the case of strip-type powder, the curves practically coincide also at the intermediate points, when the values of x_1 and λ_1 are chosen as above.

Thus, the binomial formula can be used for strip and plate type powders also in the future

$$\psi = x_1 z (1 + \lambda_1 z).$$

Similarly, we shall have for the surface ratio

$$G = 1 + 2\lambda_1 z.$$

Eliminating z from this system of equations, we get the dependence of G on ψ in the following form:

$$G = \sqrt{1 + 4 \frac{\lambda_1}{x_1} \psi}.$$

For a given value of ψ , this relationship permits a direct calculation of the corresponding value of G .

Hereafter, we shall drop the indexes of the characteristics x and λ .

4. PROGRESSIVE-BURNING POWDERS

A. General Data

In all the regressive types of powder considered here, excepting tubular powders, the surface area always diminishes when the powder is burned, because burning proceeds inside the grain in concentric layers. A tubular grain is the exception in this respect: the surface of the perforation is displaced in burning from the axis of the tube outwards, thus increasing its area, and hence partly compensates

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for the reduction of the exterior surface area. The tubular type of powder would represent a powder of constant burning area bordering between progressive and regressive forms of powder, if the tube were not to burn from its ends and remain unchanged in length.

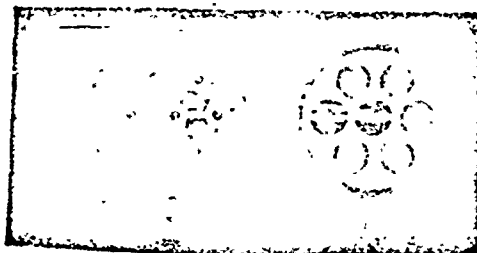


Fig. 27 - Grain with Seven Perforations:

- a - before burning;
- b - at the instant of decomposition.

At the start of burning the total surface of the tube of length $2c$ will be expressed by the following formula, if the end-areas and reduction in length are disregarded:

$$S_1 = 2\pi R 2c + 2\pi r \cdot 2c = 2\pi(R + r) \cdot 2c.$$

When the thickness burnt on the inside and the outside of the tube is e , the surface at that instant and at the same tube length will be

$$S = 2\pi(R - e)2c + 2\pi(r + e)2c = 2\pi(R + r)2c;$$

consequently, the surface area $S = S_1 = \text{const}$, because the reduction of the exterior surface is compensated for by the equivalent enlargement of the interior surface.

Under such conditions the constancy of the surface area does not depend on the diameter of the perforation. This property of the perforation surface to increase in burning indicates a means for obtaining a grain of the progressive type. This will require the use of grains with several perforations, whereby the increased surface area of the latter will compensate for the reduction of the outside area.

The grain with 7 perforations is based on this principle; the centrally disposed perforation along the axis of the grain compensates the decrease in the outside area, and the six radially disposed perforations in the vertices of a regular hexagon serve to increase the area during burning. The outside surface is located at a distance of $2e_1$ from the perforations. In such an arrangement (fig. 27a) the web thickness, i.e., the distance between the centrally located perforation and the outer perforations, as well as between the latter and the outer surface, will be the same, so that all the webs will burn simultaneously.

Burning from the perforation centers proceeds along concentric cylindrical surfaces, forming circles in section; when the latter converge and the thickness e_1 is burned in all directions, the grain disintegrates into 12 rods of irregular cross section (slivers): 6 inner, small rods and 6 outer larger rods (fig. 27b). These products of decomposition burn with a sharp reduction of the burning surface, similarly to a solid slab, and provide even greater regression because of the presence of sharp, rapidly burning projecting angles. Thus all progressive powder grains with several perforations, whose initial burning is accompanied by increased surface area disintegrate

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at some instant into regressively burning products of decomposition. This is the undesirable characteristic of powders of the progressive type.

A grain with 7 perforations usually has a standard dimension ratio: the web thickness e_1 between the perforations themselves as well as between the latter and the outer wall must be the same and equal to twice the diameter of the perforation (or the perforation diameter d must be equal to half of the web thickness e_1):

$$\underline{2e_1 = 2d;}$$

accordingly, the outside grain diameter is

$$D = 4 \cdot 2e_1 + 3d = 11d = 11e_1.$$

The length of the grain $2c$ is not great, it is usually equal to $(2-2.5)D$ or $(20-25)d$.

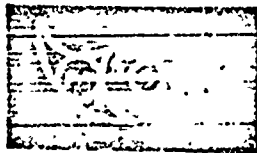


Fig. 28 - Products of Decomposition of a Grain
With 7 Perforations.

At this dimension ratio, the surface increase at the time of decomposition amounts to about 37% ($S_g/S_1 = 1.37$, where S_g is the surface being burned at the instant decomposition occurs). At the same time, if the perforations are spaced correctly, about 85% of the

grain will be burned ($\psi_s \approx 0.85$). Consequently, about 15% of the grain is burned regressively with a sharp reduction of the surface area. If the spacing of the perforations is irregular, decomposition does not take place at the same instant: webs of the least thickness are burned first, followed by gradual burning of the thicker webs; a portion of the grain undergoes progressive burning and the remaining portion suffers a sharp reduction of its area. The maximum value of S_g is smaller than in a normal grain, and this corresponds also to a lower value of ψ_s .

If we inscribe a circle in the outer prism of decomposition (fig. 28), its radius $\rho = 0.1772(d + 2e_1)$ [12]; in the case of standard dimensions $\rho = 0.1772 \cdot 3e_1 = 0.5316e_1 \approx 0.532e_1$.

The radius of the circle inscribed in the inner prisms of decomposition $\rho' = 0.0774(d + 2e_1) = 0.2322e_1 \approx 0.232e_1$. Therefore, at the end of burning of the grain as a whole, when the burning surfaces merge at the center of the outside prisms of decomposition, the thickness burned will be $e_k = e_1 + \rho = e_1 + 0.532e_1 = 1.532e_1$.

Thus the burning of progressive powders is subdivided into two sharply differing phases: 1) prior to decomposition z varies from 0 to 1, ψ varies from 0 to $\psi_s < 1$; burning proceeds with a gradually increasing area; 2) after decomposition z changes from 1 to

$$z_k = \frac{e_1 + \rho}{e_1} = 1 + \rho/e_1, \quad \psi \text{ changes from } \psi_s \text{ to } 1; \text{ burning is progressive.}$$

Some authors (V.E. Slukhotsky) consider it more expedient to consider $z_k = 1$ when $\psi = 1$; then, at the instant of decomposition, $z_s = \frac{e_1}{e_1 + \rho} < 1$ ($z_s = 0.653$ for standard dimensions).

Disintegration and regressive burning of the powder in the second phase are the defects of powders of the progressive type. In order to decrease the products of decomposition, a great many grain shapes were suggested by various authorities, of which the most interesting ones are the shapes suggested by Walsh and Kisnensky.

Walsh's grain is an improved grain with 7 perforations, whose outer wall is not a continuous (single) cylindrical surface of diameter $D = 11d$, but, rather, consists of six cylindrical surfaces circumscribed about the axis of each perforation of radius $r = \frac{d}{2} + 2e_1$ (in the case of standard dimensions $r = 2.5d = 2.5e_1$). The cross section of such a grain is as shown in fig. 29a, and at the instant of decomposition its section takes on the form shown in fig. 29b.

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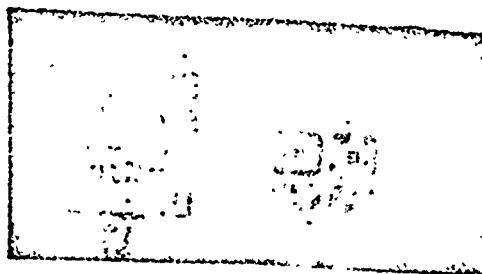
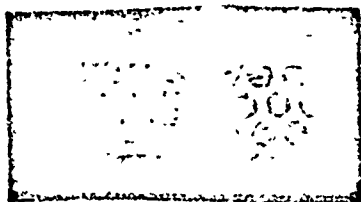


Fig. 29 - Walsh's Grain with 7 Perforations.

Fig. 30 - Burning of Kisnensky's Grain.

a) Before burning; b) at the instant of decomposition.

A grain of this type is often called a "shaped grain."

The dimensions of the products of decomposition in such a grain are considerably smaller than in an ordinary grain with 7 perforations.

Calculations show that $\psi_s \approx 0.95$, i.e., that 95% of the grain undergoes progressive burning, and only 5% is regressive. Also, for the same dimensions d , $2e_1$ and $2c$ as in a standard grain, the surface area increase is the same, or 37%, so that $G_s \approx 1.37$.

The Kisnemy grain was designed for the purpose of eliminating the products of decomposition on the one hand, and for obtaining a grain of "greater progressivity" on the other. It is in the form of a square slab with square perforations disposed in two mutually perpendicular directions (fig. 30a). According to Kisnemy, the square perforation should burn in parallel layers and retain its square form, so that burning would terminate without decomposition. Furthermore, a large number of narrow perforations should provide a highly progressive grain (according to calculations $G_K \approx 2$). This is the powder recommended by Kisnemy for obtaining extra-long-range firing; the KOCACTON (Kosartop) (Special Artillery Research Committee), headed by V.M. Trofimov, was the body engaged in the investigation of this type of powder at the time. All the theoretical work on the performance of this powder was carried out by V.A. Pashkevich.

It was found that Kisnemy's estimates were not justified either with regard to burning without decomposition, or with regard to high progressivity. Tests had shown that the square-shaped perforation does not retain its shape, because burning from the corners of the square proceeds at the same rate as along a normal to the side of the square (there is no reason to believe that the powder would burn more rapidly along the diagonal than in other directions); it proceeds from each corner in concentric circles, whose radius equals the thickness

of the burnt layer in the direction of normals to the sides of the square. As a result, the cross section is that of a square with rounded corners, and at the instant of decomposition the shape of the grain appears as in fig. 30b.

The products of decomposition amounting to 10% remain between the perforations. Hence ψ_s is not 1 but is about 0.90 (90% of the grain burns progressively). $\rho'' = (\sqrt{2} - 1)e_1$, because $e_1 + \rho$ is the diagonal of a square with side e_1 .

For reasons which will be discussed later, the powder suggested by Kisnemy did not show the high progressive quality anticipated by him.

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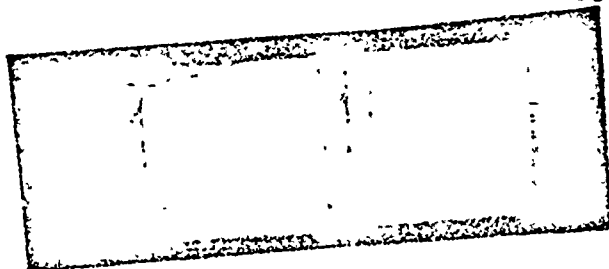


Fig. 31 - A Grain of Kisnemy's Powder Before and After Disintegration (Obtained in Firing).

Figure 31, on the left, is a Kisnemy grain with 35 perforations before burning; on the right is the same grain incompletely burned and ejected from the gun after firing. The photograph clearly shows the products of the starting disintegration having the form of the "ace of diamonds" in cross section; the perforations are almost round.

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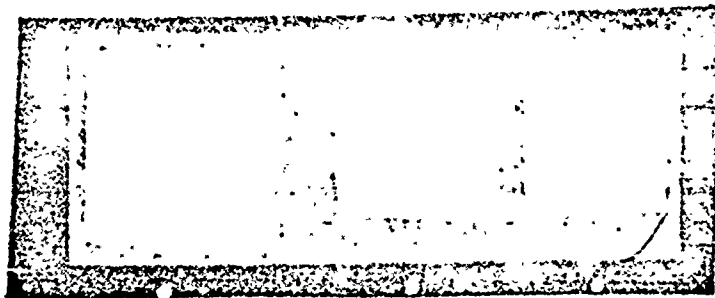


Fig. 32 - Transformation of a Square Perforation into a Round One.

Figure 32 is a large grain with a single perforation: a) before burning; b and c) intermediate states. Inasmuch as the side of the square perforation is small in comparison with the web thickness, the perforation is transformed into a circle.

B. Determination of the Characteristics of Progressive Powders

a) First phase, first method.

The law governing burning and the change of the surface of progressive type powders is expressed by the same general formulas:

$$\psi = \kappa z(1 + \lambda z + \mu z^2),$$

$$G = 1 + 2\lambda z + 3\mu z^2,$$

as for regressive types; the difference is only in the numerical values of the characteristics κ , λ and μ and in their signs ($\lambda > 0$, $\mu < 0$). The derivation of the characteristics equations is obtained similarly to regressive powders, and only the obtained expressions themselves are somewhat more complex than for regressive powders.

We shall introduce the following designations: $2c$ - grain length, $2e_1$ - web thickness; d - diameter of perforation; D - grain diameter; n - number of perforations.

The burned portion of the charge is:

$$\psi = 1 - \frac{\Lambda_{ocr}}{\Lambda_1}; \quad \frac{2e_1}{2c} = \beta;$$

$$\frac{D + nd}{2c} = \Pi_1; \quad \frac{D^2 - nd^2}{(2c)^2} = Q_1.$$

It can be easily seen that the magnitude $\Pi_1 = D + nd/2c$ is the ratio of the perimeter of the grain's cross section to a circumference whose diameter equals the length of the tube $2c$. The magnitude $Q_1 = \frac{D^2 - nd^2}{(2c)^2}$ is the ratio between the base area and the area of a circle whose diameter is $2c$.

The initial grain volume is

$$\Lambda_1 = \frac{\pi}{4}(D^2 - nd^2)2c.$$

The unburned portion of the grain by volume at a given instant is

$$\Lambda_{ocr} = \frac{\pi}{4} [(D - 2e)^2 - n(d + 2e)^2] (2c - 2e);$$

$$\frac{\Lambda_{ocr}}{\Lambda_1} = \frac{[D^2 - nd^2 - 2(D + nd)2e - (n - 1)(2e)^2]}{D^2 - nd^2} \cdot \frac{(2c - 2e)}{2c}.$$

Bearing in mind that

$$\frac{2e}{2c} = \frac{2e_1}{2c} \frac{2e}{2e_1} = \beta z$$

and dividing each term in the parenthesis by the denominator, we get:

$$\frac{\Lambda_{ocr}}{\Lambda_1} = \left[1 - \frac{2(D + nd)}{D^2 - nd^2} 2e - \frac{(n-1)}{D^2 - nd^2} (2e)^2 \right] (1 - \beta z),$$

or, substituting $\pi_1 \cdot 2c$ for $D + nd$ and $Q_1 (2c)^2$ for $D^2 - nd^2$, we have:

$$\frac{\Lambda_{ocr}}{\Lambda_1} = \left[1 - \frac{2\pi_1 \cdot 2c \cdot 2e}{Q_1 (2c)^2} - \frac{n-1}{Q_1 (2c)^2} (2e)^2 \right] (1 - \beta z) =$$

$$= \left(1 - \frac{2\pi_1 \beta z}{Q_1} - \frac{n-1}{Q_1} \beta^2 z^2 \right) (1 - \beta z) =$$

$$= 1 - \left(1 + \frac{2\pi_1}{Q_1} \right) \beta z - \left(\frac{n-1-2\pi_1}{Q_1} \right) \beta^2 z^2 + \frac{n-1}{Q_1} \beta^3 z^3;$$

$$\psi = 1 - \frac{\Lambda_{ocr}}{\Lambda_1} = \left(1 + \frac{2\pi_1}{Q_1} \right) \beta z + \frac{(n-1)-2\pi_1}{Q_1} \beta^2 z^2 - \frac{n-1}{Q_1} \beta^3 z^3 =$$

$$= \frac{Q_1 + 2\pi_1}{Q_1} \beta z \left(1 + \frac{n-1-2\pi_1}{Q_1 + 2\pi_1} \beta z - \frac{(n-1)\beta^2}{Q_1 + 2\pi_1} z^2 \right).$$

We thus get a general type formula:

$$\psi = \kappa z(1 + \lambda z + \mu z^2),$$

where

$$(*) \left. \begin{aligned} \kappa &= \frac{Q_1 + 2\pi_1}{Q_1} \beta; \\ \lambda &= \frac{n - 1 - 2\pi_1}{Q_1 + 2\pi_1} \beta; \\ \mu &= - \frac{(n - 1) \beta^2}{Q_1 + 2\pi_1}; \end{aligned} \right\} \text{ when } n = 7 \left\{ \begin{aligned} \kappa &= \frac{Q_1 + 2\pi_1}{Q_1} \beta; \\ \lambda &= \frac{2(3 - \pi_1)}{Q_1 + 2\pi_1} \beta; \\ \mu &= - \frac{6\beta^2}{Q_1 + 2\pi_1}. \end{aligned} \right.$$

These formulas show that for powders with many perforations the characteristic $\mu < 0$ and hence the convexity of curve σ , z is directed upwards (when $n = 1$, $\mu = 0$). The sign of λ depends on the difference $(n - 1) - 2\pi_1$, and in ordinary powders with 7 perforations of standard dimensions $\lambda > 0$. When the grain is shortened λ may become equal to zero: this will occur when $n - 1 - 2\pi_1 = 0$ or $n - 1 - 2\frac{D + nd}{2c} = 0$, whence

(*) These formulas are suitable not only to progressive powders where $n > 1$, but also for regressive shapes - solids of revolution, for example: for $n = 1$ (tube) and $n = 0$ (round slab without perforations).

Hence, they are general formulas: thus, for example, for a tube when $n = 1$, $\lambda < 0$, $\mu = 0$, when $n = 0$ (solid slab) $\lambda < 0$, $\mu > 0$, as was established in the previous formulas.

$$\frac{D + nd}{2c} = \frac{n - 1}{2}.$$

Under such conditions, perforated powder is no longer progressive.

In particular, for a grain of standard cross section with 7 perforations, whose $D = 11d$, the condition $\lambda = 0$ is satisfied when

$$2c = 6d = 3 \cdot 2e_1,$$

and since $\mu < 0$, the area of such a short grain will diminish when burned.

A study of the λ coefficient shows that progressivity increases with the number of perforations, if the diameter of the perforations at the same web thickness decreases and the length of the slab increases.

The expression for the surface (area) change will have the following general form:

$$G = 1 + 2\lambda z + 3\mu z^2.$$

In the case of rectangular shapes, such as the Kisnensky grain, the magnitude π_1 is the ratio between the perimeter of the cross section and the perimeter of a square with side $2c$ equal to the length of the slab, and Q_1 is the ratio between the cross-sectional area of the grain and the area of the same square with side $2c$.

If we call the side of a square slab A_1 , the side of the square perforation a_1 , the length of the slab $2c$ and the number of perforations n^2 (n horizontal and vertical rows),

$$\Pi_1 = \frac{(A_1 + n^2 a_1)}{2c}, \quad Q_1 = \frac{A_1^2 - n^2 a_1^2}{(2c)^2}, \quad \beta = \frac{2e_1}{2c}.$$

These formulas hold true for Walsh's grain as well, but inasmuch as the cross section of this grain is more complex, the formulas for Π_1 and Q_1 are likewise more complex.

The outside wall of Walsh's grain can be considered consisting of six arcs whose lengths equal 1/3 of a circumference of diameter $d_1 = d + 2 \cdot 2e_1$ and which are described from the centers of six perforations, and six arcs each measuring 1/6 of a circumference of diameter d_1 described from the outside vertices of equilateral triangles with side $a_1 = 2e_1 + d_1$, whose other two vertices lie in the center of the perforations (fig. 33).

In such a case the cross-sectional area S_T of the grain consists of the following elements:

1) 12 triangles with side a_1 less three sectors of a circle of diameter d_1 at their apexes;

2) six sectors each measuring 1/3 of a circle of diameter d_1 less six sectors each equivalent to 1/3 of a circle of diameter d_1 :

$$S_T = 12 \left(\frac{\sqrt{3}}{4} a^2 - 3 \frac{1}{6} \frac{\pi}{4} d_1^2 \right) + 6 \frac{1}{3} \frac{\pi}{4} (d_1^2 - d_1^2) - \\ - \frac{\pi}{4} \left(\frac{12\sqrt{3}}{\pi} a^2 - 8d_1^2 + 2d_1^2 \right).$$

The perimeter P of the grain will consist of:

- 1) six arcs each measuring $1/3$ of a circumference of diameter δ_1 ;
- 2) seven circles of diameter d_1 ;
- 3) six arcs each measuring $1/6$ of a circumference of diameter d_1 (in the angles included in the outside web).

$$P = \pi \left(6 \frac{1}{3} \delta_1 + 7d_1 + 6 \frac{1}{6} d_1 \right) = \pi(2\delta_1 + 8d_1) = 2\pi(\delta_1 + 4d_1).$$

Then:

$$\beta = \frac{2e_1}{2c_1};$$

$$\Pi_1 = \frac{P}{\pi 2c_1} = \frac{2(\delta_1 + 4d_1)}{2c_1};$$

$$Q_1 = \frac{s_T}{\frac{\pi}{4}(2c_1)^2} = \frac{\frac{12\sqrt{3}}{\pi}a_1^2 + 2\delta_1 + 8d_1^2}{(2c_1)^2}.$$

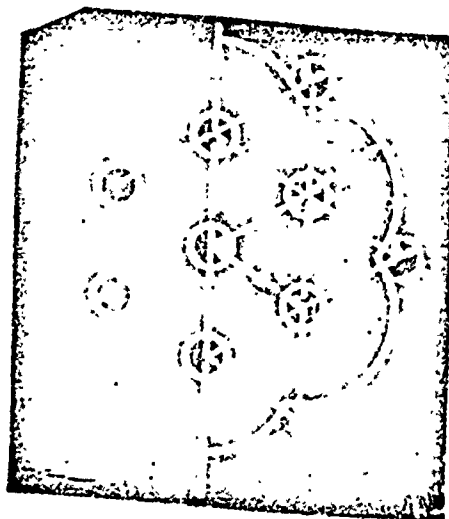


Fig. 33 - Diagram of Walsh's Grain.

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b) A binomial formula for the second phase.

Since for a standard grain with 7 perforations the characteristic μ is small, the law governing burning $\psi = f(z)$ can also be expressed with sufficient accuracy by means of a binomial formula:

$$\psi = \kappa_1 z(1 + \lambda_1 z).$$

The characteristics κ_1 and λ_1 will be found under the condition that when $z = 1$, $\psi = \psi_s$, and when $z = 0.5$, the value of ψ according to a trinomial formula would be equal to the value of ψ according to the binomial one. We thus obtain a system of two equations as for regressive powders:

when $z = 1$

$$\kappa(1 + \lambda + \mu) = \psi_s = \kappa_1(1 + \lambda_1);$$

when $z = 0.5$

$$\frac{\kappa}{2} \left(1 + \frac{\lambda + \mu}{2} \right) = \frac{\kappa_1}{2} \left(1 + \frac{\lambda_1}{2} \right).$$

Solving it we get

$$\kappa_1 = \kappa \left(1 - \frac{\mu}{2} \right),$$

and then

$$\lambda_1 = \frac{\psi_s}{\kappa_1} - 1$$

(instead of $\lambda_1 = \frac{1}{\kappa_1} - 1$ for regressive powders).

Example. Compute the shape characteristics of progressive powders.
Grain with 7 perforations.

$$2e_1 = 1; d = 0.5; D = 5.5; 2c = 12.5 \text{ mm}$$

$$\beta = \frac{2e_1}{2c} = \frac{1}{12.5} = 0.08; \quad \pi_1 = \frac{D + 7d}{2c} = \frac{9}{12.5} = 0.720;$$

$$Q_1 = \frac{D^2 - 7d^2}{(2c)^2} = \frac{30.25 - 7 \cdot 0.25}{156.25} = \frac{28.50}{156.25} = 0.1824;$$

$$Q_1 + 2\pi_1 = 0.1824 + 2 \cdot 0.720 = 1.6224;$$

$$\kappa = \frac{Q_1 + 2\pi_1}{Q_1} \beta = \frac{1.6224}{0.1824} \cdot 0.08 = 0.712;$$

$$\lambda = \frac{(n - 1) - 2\pi_1}{Q_1 + 2\pi_1} \beta = \frac{6 - 1.44}{1.622} \cdot 0.08 = 0.225;$$

$$\mu = - \frac{(n - 1)\beta_1^2}{Q_1 + 2\pi_1} = - \frac{6 \cdot 0.0064}{1.622} = - 0.0237;$$

$$\psi_s = \kappa(1 + \lambda + \mu) = 0.712(1 + 0.225 - 0.0237) = 0.712 \cdot 1.2013 = 0.855;$$

$$\epsilon_s = 1 + 2\lambda + 3\mu = 1 + 0.45 - 0.0711 = 1.379.$$

In the case of the binomial formula $\psi = \kappa_1 z(1 + \lambda_1 z)$:

$$\kappa_1 = \kappa \left(1 - \frac{\mu}{2}\right) = 0.712 \cdot \left[1 - \left(-\frac{0.0237}{2}\right)\right] = 0.712 \cdot 1.0118 = 0.720;$$

$$\lambda_1 = \frac{\psi_s}{\kappa_1} - 1 = \frac{0.855}{0.720} - 1 = 0.1873;$$

$$\psi_s = \kappa_1(1 + \lambda_1) = 0.720(1 + 0.1873) = 0.855;$$

$$\epsilon_s = 1 + 2\lambda_1 = 1.375.$$

It can be seen that the difference between the values of ϵ_s corresponding to the instant of disintegration obtained by trinomial and binomial formulas would be very small (0.004), so that in practice the binomial formula can be used for powders with 7 perforations in the first phase of burning.

c) Second phase (after disintegration).

The products obtained after decomposition in the form of small prisms of triangular cross section (with curved sides) burn regressively with rapid surface reduction, similarly to a square or round slab.

A detailed investigation made by G.V. Oppokov [13], using the assumption that burning proceeds in strictly parallel layers, provides general expressions: one - prior to burning of the thinner inside prisms, and the other - until the end of burning of the outside prisms. Using the formulas suggested by him as a basis, Oppokov

prepared a table for the values of ψ as a function of z when the small and large prisms are burned and then when only the remainders of the large prisms are burned. An analysis of his data shows that the change in the surface area of the products of decomposition almost corresponds to the change in the surface of a cube, i.e., its burning is more regressive than in the case of a prismatic slab.

The binomial formula can be used for determining the relationship ψ, z in the second phase. Transferring the origin of the coordinates to the point of decomposition ($z_s = 1, \psi = \psi_s$) we will have

$$\psi - \psi_s = \kappa_2(z - 1) [1 + \lambda_2(z - 1)] \dots, \quad (30)$$

with z varying from 1 to z_K .

We shall determine κ_2 and λ_2 by imposing the following requirements:

- 1) when $z = z_K$, ψ must equal unity according to formula (30);
- 2) at the end of burning when $z = z_K$ the surface ϵ must become equal to zero.

Differentiating equation (30) with respect to z , we get:

$$\frac{d\psi}{dz} = \frac{S}{\Lambda_1} e_1 = \kappa_2 [1 + 2\lambda_2(z - 1)].$$

The second requirement will be satisfied if:

$$1 + 2\lambda_2(z_K - 1) = 0.$$

The first requirement will be written thus:

$$1 - \psi_s = \kappa_2(z_K - 1) [1 + \lambda_2(z_K - 1)].$$

Solving this equation we find that

$$\lambda_2 = \frac{-1}{2(z_K - 1)}; \quad \kappa_2 = \frac{2(1 - \psi_s)}{z_K - 1}.$$

Calculations by means of these formulas show that for a standard grain with 7 perforations when $z_K = 1.532$ $\psi_s = 0.855$:

$$\lambda_2 = \frac{1}{2 \cdot 0.532} = -0.94; \quad \kappa_2 = \frac{2 \cdot 0.145}{0.532} = 0.545;$$

for Walsh's grain when $z_K = 1.232$ and $\psi_s = 0.95$

$$\lambda_2 = \frac{-1}{2 \cdot 0.232} = -2.16; \quad \kappa_2 = \frac{2 \cdot 0.05}{0.232} = 0.432.$$

C. Graphic Representation of Relations $\psi - z$, $\frac{S}{S_1} - z$, $\frac{S}{S_1} - \psi$ for Progressive Shapes

In order to construct diagrams $\psi - z$; $\frac{S}{S_1} - z$; $\frac{S}{S_1} - \psi$ for progressive shapes, we shall resort to the following general formulas:

$$\psi = \kappa z(1 + \lambda z + \mu z^2) \quad \text{and} \quad \frac{S}{S_1} = 1 + 2\lambda z + 3\mu z^2,$$

which are applicable also to these shapes, but now when $z = 1$

$\psi = \psi_s < 1$; the end of burning occurs when $z > 1$, the coefficient $\mu < 0$ (not only for grains with 7 perforations, but also for the Kisnensky grain.)

Accordingly, the diagrams will appear as shown in fig. 34 and 35:

- I - grain with 7 perforations;
- II - grain with shaped outer web;
- III - Kisnensky's grain.

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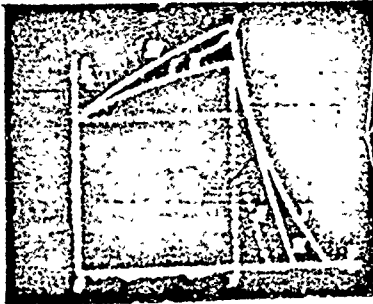


Fig. 34 - $\epsilon = f(z)$ relationship for progressive grains.

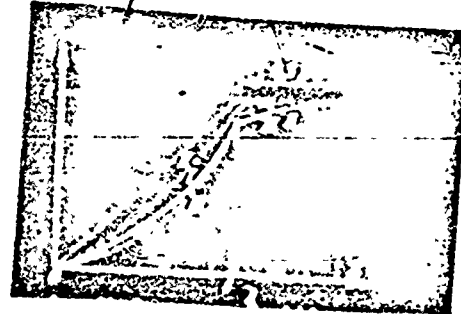


Fig. 35 - $\psi = f_1(z)$ relationship for progressive grains.

Inasmuch as $\mu < 0$, the convexity of the $S/S_1, z$ curves is directed upwards. When constructing an S/S_1 diagram as a function of ψ , the corresponding points of the $S/S_1, z$ diagram will be displaced to the left (the reverse of regressive grains), this displacement being the smaller the greater is the value of z ; hence the $S/S_1, \psi$ curves will likewise remain convex upwards as do the $S/S_1, z$ curves (fig. 36).

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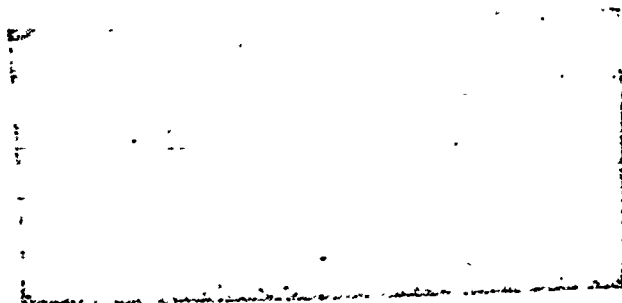


Fig. 36 - $G = f_2(\psi)$ Relation for Progressive Powders.

a) Inhibited powders of high progressivity

Inhibited powders constitute one of the forms of highly progressive powders.

These powders first appeared in Russia soon after the appearance of Kisnemy's powders and were developed jointly with the latter. This problem was studied by O.G. Filippov, an instructor at the Artillery Academy, in 1920-1928.

Inhibited powder is obtained from ordinary tubular powder, whose outside surface is coated with a special nonburning substance.

When such a powder is ignited, only the inside surface of the grain burns, which surface increases in proportion to the diameters ratio $D_0 : d_0$. When the diameter of the perforation equals the thickness of the tube

$$\frac{S_K}{S_1} = \frac{D_0}{d_0} = 3.$$

The resulting progressiveness is greater than in a Kisnemy grain with 36 perforations.

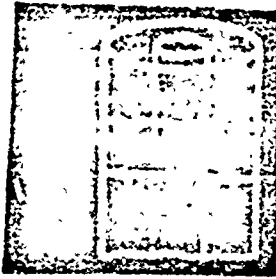


Fig. 37 - Inhibited Tubular Powder.

Notwithstanding the apparent simplicity of this idea and the theoretical possibility of obtaining high progressivity, the practical realization of same was found to be very difficult, because the inhibitor layer must not burn and must be capable of protecting the outer tube surface from burning. At the same time it must be sufficiently strong to withstand abrasion when shaken and be strong enough not to be torn off the surface by the action of the gases.

The undesirable property of inhibited powders is greater smoking when fired, due to the disintegration of the inhibitor at the instant the shell is ejected from the bore of the gun.

b) Characteristics of inhibited powders.

The outside surface and ends of the powder are inhibited and do not burn.

In contradistinction to an ordinary tube, the web thickness of an inhibited powder burns in one direction only and should therefore be denoted by e_1 rather than by $2e_1$, which is the usual designation.

Applying the general method, we get:

$$\Lambda_1 = \frac{\pi}{4} [(d_0 + 2e_1)^2 - d_0^2] 2c = \frac{\pi}{4} 2c [2d_0 2e_1 + 4e_1^2] = \pi 2c (d_0 e_1 + e_1^2);$$

$$\Lambda_{ocr} = \frac{\pi}{4} [(d_0 + 2e_1)^2 - (d_0 + 2e)^2] 2c = \pi 2c [d_0 e_1 + e_1^2 - (d_0 e + e^2)];$$

$$\frac{\Lambda_{ocr}}{\Lambda_1} = 1 - \frac{d_0 e + e^2}{d_0 e_1 + e_1^2} = 1 - \frac{\frac{e}{e_1} + \frac{e_1}{d_0} \left(\frac{e}{e_1}\right)^2}{1 + \frac{e_1}{d_0}} = 1 - \frac{z + \frac{e_1}{d_0} z^2}{1 + \frac{e_1}{d_0}};$$

$$\psi = 1 - \frac{\Lambda_{ocr}}{\Lambda_1} = \frac{1}{1 + \frac{e_1}{d_0}} z \left(1 + \frac{e_1}{d_0} z\right);$$

Comparing it with the usual formula $\psi = \lambda z(1 + \lambda z)$, we get:

$$\lambda = \frac{e_1}{d_0}, \quad \lambda = \frac{1}{1 + \lambda} = \frac{1}{1 + \frac{e_1}{d_0}};$$

$$\sigma = \frac{S}{S_1} = 1 + 2\lambda z = 1 + \frac{2e_1}{d_0} z; \quad \sigma_K = 1 + \frac{2e_1}{d_0} = \frac{D_0}{d_0}.$$

When the tube thickness e_1 equals its diameter d_0 ,

$$\lambda = 1; \kappa = \frac{1}{1 + \lambda} = 0.5; G_K = 1 + 2 = 3.$$

By reducing the diameter of the perforation with the web thickness remaining the same, the geometric progressiveness can be considerably increased:

$$d_0 = \frac{e_1}{2}; \lambda = 2; \kappa = \frac{1}{3}; G_K = 1 + 2\lambda = 5.$$

CHAPTER V - BURNING RATE.

The burning rate of powder mainly depends on its properties and temperature, and the pressure and temperature of the gases surrounding it.

Inasmuch as the temperature of the gases formed during burning of powder as yet does not lend itself to experimental determination, the burning rate is usually expressed as a function of its properties and gas pressure which is known from experiment at any given instant of time.

The functional dependence of the burning rate u on pressure of the form $u = f(p)$ is known as the "burning rate law," and this law is expressed by various empirical formulas as given by different authors.

Experimental determination of the burning rate of powder is possible on the basis of a test curve depicting pressure as a function of time; this involves the use of the fundamental relationship of the geometrical law of burning giving the relation between ψ and $z = e/e_1$.

For determining the burning rate, use is usually made of strip, plate or tubular powder, of uniform thickness; its dimensions are carefully measured, the mean values of $2e_1$, $2b$, $2c$ or $2e_1$, D_0 , d_0 , $2c$ are determined, the characteristics of κ and λ are calculated (using the binomial formula), and a graph is constructed for $\psi = f(z)$.

The powder is then burned in a manometric bomb using a strong igniter, to insure simultaneous ignition along the entire powder surface so as not to impair the initial dimensions used for calculating the κ and λ characteristics.

The analysis of the bomb test data is conducted in the following manner.

Having obtained from the bomb test a curve of pressure p as a function of time t , and upon determining for this powder on the basis of the derived relationships the law governing the variation of ψ with z or e and constructing the corresponding ψ , z or ψ , e diagram, we can determine the rate of burning u at the given pressure. Indeed, knowing the values of p , we can determine by means of the general pyrostatics formula or from tables the values of ψ , for which we take the values of e from the ψ , e diagram. The difference between the neighboring values of e will give the increment Δe for the time interval Δt , known from measurement of the p , t curve; the $\Delta e / \Delta t$ ratio gives the burning rate u at p which is an average value for the given section. Thus, having at our disposal calculated data in the form of a table, we can determine the variation in the burning rate and of the given powder due to change of pressure p (Table 12).

Table 12 - Determining the Burning Rate $u = f(p)$

Auxiliary table or diagram compiled on the basis of the geometric law of burning			Table Derived from Test Data						
z	ψ	e	1	2	3	4	5	6	7
			t	p	p_{cp}	ψ	According to Auxiliary Diagram	Δe	$\frac{\Delta e}{\Delta t} = u$
0	0	0	Q	p_B	$\frac{1}{2}(p_B + p')$	0	0	$\Delta e' = e' - 0$	u'
z'	ψ'	e'	t'	p'	$\frac{1}{2}(p' + p'')$	ψ'	e'	$\Delta e'' = e'' - e'$	u''
z''	ψ''	e''	t''	p''		ψ''	e''		
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.		
1	1	e_1	t_K	p_m		1	e_1		

Plotting the obtained values of u , p_{cp} ($p_{cp} = p_{mean}$ - Translator) on a graph, we can find the change of u with change of pressure p , and thus determine the burning rate law.

The formulas most often used for determining the burning rate are the following.

a) Vieille's formula (exponential equation)

$$u = Ap^{\nu},$$

where A and ν depend on the nature of the powder, and, in particular, ν may be equal to unity.

The smaller the value of ν , the less sensitive is the powder to pressure changes.

For smokeless powders Vieille used $\nu = 2/3$, and for ordinary black powders $\nu = 1/2$. Our tests with slowly burning black powders for a time fuze gave the value of $\nu = 1/5$.

G.A. Zabudsky used $\nu = 0.93$ for pyroxyline powders. Some authors use $\nu = 1$ for cordites and $\nu = 1.07$ for ballistite.

b) Binomial formula

$$u = a + bp.$$

It was first used by Prof. S.P. Vukolov (1891-1897) in the Naval Technical Laboratory, and then by Wolf (1903) and Prof. I.P. Gravé (1904). Muiraur employed this formula considerably later (1930-1935).

In his thesis (1904) I.P. Gravé [14] compared formula $u = Ap^{\nu}$ with formula $u = ap + b$ by analyzing a large number of bomb tests conducted by himself and others and arrived at the following conclusion: "Both formulas can be considered equally valid for expressing the law governing the change of burning rate under varying pressure, because the mean errors obtained with the use of these formulas are

generally the same, and both formulas give practically identical results."

This deduction which appears strange at first glance, namely, that a parabola (semicubical) and a straight line not passing through the origin give identically accurate results, can be explained as follows.

The first tests in bombs since the year 1880 were conducted with the use of cylindrical crushers which do not permit recording pressures below 300-400 kg/cm² (fig. 38). The test data (points) are usually scattered to a certain extent and do not lie on a definite line. As a result, some of the investigators drew a parabola $u = Ap^v$ through these points, and others drew a straight line $u = ap + b$ which does not pass through the origin of the coordinates (curve 2).

The conclusion arrived at by Prof. Gruvé confirms the fact that both lines pass sufficiently close to the points plotted on the basis of tests at pressures exceeding 400 kg/cm².

The relationship $u = f(p)$ could not be obtained experimentally at low pressures (< 400 kg/cm²) at that time, and the question of the true relationship continued to remain open.

c) Formula $u = Ap$.

Charbonier (1908) first accepted the burning rate law in its general form $u = Ap^v$, and then, on the basis of his own analysis of test curves p, t obtained in bomb tests, arrived at the conclusion that for French strip-type B powders v can be taken equal to $v = 1$.

This law was accepted also by our Prof. N.F. Drozdov in his thesis 157 in the year 1910.

In 1913, Schmitz burned tubular powders in a large Krupp bomb, using an elastic bar with an optical method of recording pressures instead of a crusher. He succeeded in obtaining a full curve of the pressure increase in the bomb from the start to the end of powder burning. In order to evaluate the accuracy of either burning rate law, he introduced a new criterion, and proved by means of an actual test the justness of the burning rate law in the form $u = Ap$.

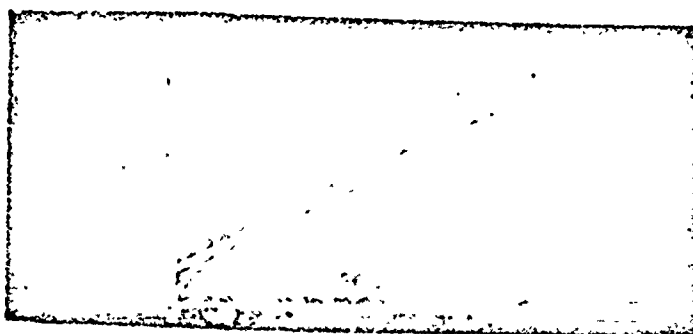


Fig. 38 - Dependence of Burning Rate on Pressure.

The criterion determining the justness of this law confirmed by experiment is presented below.

We shall assume that the following law holds true:

$$u = Ap;$$

since the burning rate is a ratio between the increment of the burned thickness de and the corresponding time element dt , i.e.,

$$u = \frac{de}{dt},$$

then

$$de/dt = Ap;$$

$$de = Ap dt.$$

Integrating, we get:

$$\int_0^{e_1} de = e_1 = A \int_0^{t_k} p dt \text{ or } \int_0^{t_k} p dt = \frac{e_1}{A}.$$

Burning terminates when thickness e_1 is burned. The full time of burning will be t_k , and we will have:

$$e_1 = A \int_0^{t_k} p dt,$$

whence

$$\int_0^{t_k} p dt = \frac{e_1}{A} = \text{const (for the given powder)}.$$

Magnitudes e_1 and A characterize the dimensions and nature of the powder and do not depend on the conditions of loading.

It follows: that if the burning rate law $u = Ap$ is correct, the pressure impulse of the powder gases depends only on the burned thickness of the powder, on the burning rate coefficient, and on the characteristic properties of the powder, and does not depend on the loading density.

The full pressure impulse during the full time of burning equals half of the thickness e_1 of the burning layer divided by the burning rate coefficient A , and does not depend on the loading density.

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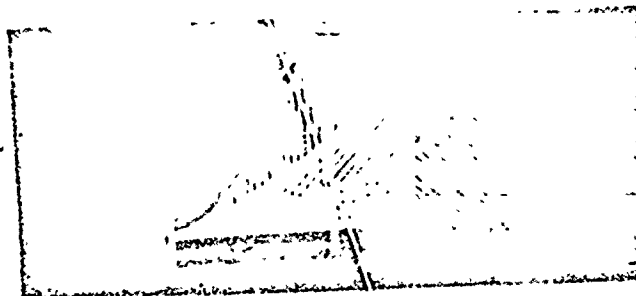


Fig. 39 - p, t Curves for Different Loading Densities.

When conducting tests with tubular powder of one kind in a Krupp bomb, with Δ varying from 0.12 to 0.26, and upon measuring the area under the pressure curves $p = f(t)$, Schmitz had found that the areas $\int_0^{t_K} p dt$ found by experiment are actually equal to one another, which finding confirms the validity of the $u = Ap$ law.

Figure 39 shows the form and arrangement of p, t curves.

The greater the loading density, the smaller is the burning time and the higher is the gas pressure p, t curve.

Were the $u = de/dt = ap + b$ law valid, then, after transformation, we would have:

$$de = apdt + bdt$$

and

$$\int_0^{t_K} p dt = e_1/a - \frac{b}{a} t_K.$$

Inasmuch as t_K decreases when Δ increases, then, according to the $u = ap + b$ law, the full gas pressure impulse should become greater with increase of loading density.

When applying the $u = Ap^\nu$ law, where $\nu < 1$, an analogous

deduction is obtained.

Schmitz's tests have shown that $\int_0^{t_K} p dt$ does not depend on the

loading density, which served to prove the validity of the $u = Ap$ law.

These tests had attracted the attention of many investigators and provided data for the verification of theoretical deduction and investigations. The latter include the work of Muiraur on the study of the burning rates of colloidal powders (1927-1928) and of calculating the amount of heat transferred to the walls during burning of powder (1924-1925).

M.E. Serebriakov's tests with pyroxyline and nitroglycerine powders confirmed the validity of the $u = Ap$ law. These tests will be discussed in greater detail in section III dealing with the physical law of burning.

Thus it may be assumed that the value of u as the rate of penetration of the burning reaction inside the grain is directly proportional to pressure, i.e., it is expressed by the formula:

$$u = Ap.$$

Here A can be expressed as the ratio between the burning rate at $p = 1$ (we shall designate it by u_1) and the magnitude of this pressure $p = 1$:

$$A = u_1/1$$

(the subscript "1" indicates that this burning rate refers to pressure $p = 1$).

This formula can be rewritten thus:

$$u = \dot{u}_1 \cdot \frac{p}{1} = u_1 p;$$

when $p = 1$ $u = u_1$.

The dimensionality of the value of u_1 can be seen from the equality

$$u_1 = \frac{u}{p} \frac{dm}{sec} : \frac{kg}{dm^2},$$

i.e., this represents the rate referred to unit pressure.

Similarly to powder energy f and covolume α , the magnitude u_1 constitutes a fundamental ballistic characteristic of powder and, similarly to f and α , depends on the physical and chemical properties of the powder.

The value of u_1 for pyroxyline powders varies from 0.0000060 to 0.0000090 dm/sec : kg/dm². The thicker the powder, the greater is the content of volatiles and the slower is the burning of the powder. The higher the nitrogen content in pyroxyline, the more rapid is the burning. In nitroglycerine powders u_1 depends in the main on the nitroglycerine content itself, and the greater its content the more rapid is the burning. The admixture of dinitro-derivatives in powders in a nonvolatile solvent usually reduces the burning rate.

Varying of the content of volatiles by +1% lowers the burning rate of pyroxyline powders by 10-12%.

The following empirical formula for determining the burning rate of pyroxyline powders, introduced for the first time by

N.F. Drozdov [16] in our country, appears in American literature:

$$u_1 = 10^{-4} \frac{0.0025\varepsilon}{48(180 - t^0) + 2561h + 908.5h'}$$

where $\varepsilon = 69,400(N - 6.37)$ - powder energy in kg-m/kg;

t^0 - temperature of powder;

h - content of volatile substances in %, removed by six hours of drying (moisture);

h' - content of residual solvent in % - not removed after six hours of drying;

N - nitrogen content in %.

This formula clearly shows the effect of various individual factors on the burning rate, but does not give sufficient satisfactory results as regards our own domestic powders.

The following formula is better adopted to our pyroxyline powders and is more convenient for performing the necessary calculations:

$$u_1 = \frac{0.175(N - 6.37)}{0.04(220 - t^0) + 3h + h'} \frac{\text{mm}}{\text{sec}} : \frac{\text{kg}}{\text{dm}^2} =$$

$$= \frac{0.175 \cdot 10^{-4}(N - 6.37)}{0.04(220 - t^0) + 3h + h'} \frac{\text{dm}}{\text{sec}} : \frac{\text{kg}}{\text{dm}^2},$$

where 220 is the ignition temperature of the powder;

$$\varepsilon = 700,000(N - 6.37 \text{ kg-dm/kg}).$$

Letan assumed on the basis of the kinetic theory of gases that the burning of powder is a process in which the powder molecules are split by the impact of gas molecules, and offers the following formula for determining u_1 :

$$u_1 = \frac{g}{\delta \sqrt{\pi c_p}} \left(1 + \frac{c_1^2}{c_p^2} \right) e^{\frac{c_1^2}{c_p^2}},$$

where g = acceleration of gravity;

δ = physical density of powder;

c_p = probable velocity of molecules of gases formed in burning of powder;

c_1 = velocity of active gas molecules, whose kinetic energy is sufficient to split off at least one molecule when the surface of the powder undergoes an impact.

c_p and c_1 depend on the nature of the powder and of the gases formed during its combustion.

Schmitz had conducted his tests at loading densities Δ of from 0.12 to 0.26. Later tests had shown that at very low loading densities ($\Delta \geq 0.015$) the integral $\int_0^{t_K} p dt$ is a linearly decreasing function of time:

$$\int_0^{t_K} p dt = S_0 - \pi t.$$

As was shown above, such a relationship is obtained under the burning rate law $u = sp + b$.

In the tests conducted by M.E. Serebriakov [5] and A.I. Kokhanov it was shown that the integral $\int_0^{t_K} p dt$ changes with increase of the time of burning only in the case of powders of considerable thickness ($2e_1 > 0.5$); in the case of very thin powders, the full impulse, even at low loading densities, does not depend on the loading density. This shows that the speed of the process governing the heating of the entire powder mass is of importance, and that the increase of the powder temperature increases the burning rate u and reduces the value of the integral

$$\int_0^{t_K} p dt = e_1 / u_1.$$

In order to determine the effect of heating on the burning rate of powder under a given constant pressure, tests were conducted with powder strips burned at different temperatures in open air. It was found that the time of burning of a strip of a given length varies from 14.1 seconds at $t = 15^\circ\text{C}$ to 9.4 seconds at $t = 50^\circ\text{C}$, and hence the rate of burning increases 1 1/2 times. The integral $\int p dt = p_a \cdot t_K$ was reduced in the same proportion, where p_a is atmospheric pressure.

The effect of heating on the burning rate of powder can be confirmed by the following tests.

If several charges of the same density are burned in succession in a bomb without cooling, the latter becomes quite hot. The powder inside the bomb becomes heated also, because the time between charging and the end of burning is considerable (several minutes). The value of the integral becomes smaller with each successive test, which

condition points at an increasing rate of burning u_1 for the same powder thickness.

These tests permit the conclusion that the reduction of the integral I_K with decrease of Δ for thick powders when $\Delta < 0.10$ is the result of heating of the powder mass under the condition of relatively slow burning, whereby the degree of heating and hence the increase in the value of u_1 is the greater, the smaller the value of Δ , i.e., the slower is the burning of the powder at low pressures.

Inasmuch as the integral of I_K decreases with the decrease of Δ , the above will be theoretically valid if the burning rate laws $u = Ap^v$ and $u = ap + b$ are adhered to. Tests conducted by M.E. Serebriakov (1932) with powders with hard solvents showed that at pressures $p_m > 1000 \text{ kg/cm}^2$ the linear law $u = Ap$ can be applied to determine the burning rate, and that at pressures $p_m < 1000 \text{ kg/cm}^2$ the law expressed by the formula $u = A_1 p^{0.82}$ will apply.

In analyzing later tests conducted by Prof. Yu. A. Pobedonostsev in bombs with nozzles at very low pressures (5 to 250 atm), Prof. Ya.M. Shapiro arrived at the relationship $u = 0.37 p^{0.7}$ which, seemingly, is contradictory to the relationship $u = Ap$.

Actually, as was shown above, the decrease of the integral $\int p dt$ at small values of Δ can be explained also when applying the $u = u_1 p$ law by the increase of the burning rate u_1 due to heating of the powder. This explanation is founded on the theory of Prof. Ya.B. Zeldovich mentioned earlier.

In any case a more accurate evaluation of either expression for the burning rate law requires further investigations (see Section III).

Inasmuch as powders in gun barrels burn under high pressures and under high loading densities, the following burning rate law may be considered valid for such powders:

$$u = u_1 p.$$

Going back to the formula for expressing the rate of gas formation, we can now write it as follows:

$$\frac{d\psi}{dt} = \frac{S_1}{\Delta_1} \frac{S}{S_1} u_1 p \quad (31)$$

or

$$\frac{d\psi}{dt} = \frac{\kappa}{e_1} u_1 \frac{S}{S_1} p = \frac{\kappa}{I_K} \epsilon p, \quad (32)$$

where $\frac{S_1}{\Delta_1}$ and $\frac{S}{S_1}$ depend on the geometry of the powder;

u_1 is the burning rate of powder when $p = 1$; it characterizes the nature of the powder and the degree to which it is heated;

p is the pressure at which the powder is burned; it characterizes the influence of the surrounding medium on the powder and depends on $\Delta, f, \alpha, \delta, \psi$.

CHAPTER VI - PRESSURE VARIATION AS A FUNCTION OF TIME

We have derived above the following formulas: a) a formula for determining the rate of gas formation

$$\frac{d\psi}{dt} = \frac{S_1}{\Delta_1} \frac{S}{S_1} u_1 p = \frac{\kappa}{e_1} u_1 \epsilon p = \frac{\kappa}{I_K} \epsilon p \quad (33)$$

and b) the general pyrostatics formula which takes the igniter into account

$$p = p_B + \frac{f \Delta \psi}{1 - \frac{\Delta}{\delta} - \Delta \left(\alpha - \frac{1}{\delta} \right) \psi} = p_B + \frac{f \Delta \psi}{\Lambda_\psi}, \quad (34)$$

where $\Lambda_\psi = 1 - \frac{\Delta}{\delta} - \Delta \left(\alpha - \frac{1}{\delta} \right) \psi = \frac{W_\psi}{W_0}$ is the relative free space in the bomb in which the powder is burned.

It is necessary to determine the dependence of the pressure change on time when the powder is burned in a constant volume, i.e., to give an analytical expression for the relationship between $p = f(t)$, dp/dt and the full time of burning t_K .

Differentiating equation (34) with respect to t , we get after simple transformations:

$$\frac{dp}{dt} = \frac{f \Delta \left[\Lambda_\psi + \Delta \left(\alpha - \frac{1}{\delta} \right) \psi \right]}{\Lambda_\psi^2} \frac{d\psi}{dt} = \frac{f \Delta}{(1 - \alpha \Delta)} \frac{\left(1 - \frac{\Delta}{\delta} \right) (1 - \alpha \Delta)}{\Lambda_\psi^2} \frac{d\psi}{dt}.$$

$1 - \frac{\Delta}{\delta}$ is the value of Λ_ψ at the start of burning;

$1 - \alpha \Delta$ the same at the end of burning,

$$1 - \frac{\Delta}{\delta} > \Lambda_\psi > 1 - \alpha \Delta.$$

It may be assumed with sufficient accuracy for practical purposes that when $\psi_{cp} = 1/2$

$$K_{\psi_{cp}} = \frac{\left(1 - \frac{\Delta}{\delta}\right) (1 - \alpha \Delta)}{\Lambda_{\psi_{cp}}^2} \approx 1.$$

Then, making use of the relation (33), we get:

$$\frac{dp}{dt} = \frac{f\Delta}{1 - \alpha\Delta} \frac{d\psi}{dt} = \frac{f\Delta}{1 - \alpha\Delta} \frac{u_1}{e_1} \kappa_{ep}. \quad (35)$$

In order to simplify further derivations, we shall consider a powder whose surface area changes little when burned, so that it may be assumed that $G = G_{cp} = \text{const.}$

To such powders belong the tube and the strip, for which the binomial relationship $\psi = \kappa z(1 + \lambda z)$ is valid, whereby for the end of burning ($z = 1, \psi = 1$)

$$1 = \kappa(1 + \lambda);$$

$$G_{cp} = \frac{1 + 1 + 2\lambda}{2} = 1 + \lambda.$$

Therefore

$$\kappa G_{cp} = \kappa(1 + \lambda) = 1,$$

and for tubular or (to a lesser degree) strip powder

$$\frac{dp}{dt} = \frac{f\Delta}{1 - \alpha\Delta} \frac{u_1}{e_1} p = \frac{p_m - p_B}{I_K} p. \quad (36)$$

We shall introduce the designation

$$\tau = \frac{e_1}{u_1} \frac{1 - \alpha \Delta}{f \Delta} = \frac{I_K}{P_m - P_B} \text{ [sec]}; \quad (37)$$

then

$$\frac{dp}{dt} = \frac{p}{\tau}. \quad (38)$$

Upon separating the variables:

$$\frac{dp}{p} = \frac{dt}{\tau}.$$

Integrating, we get

$$\int_{P_B}^p \frac{dp}{p} = \frac{1}{\tau} \int_0^t dt \quad \text{or} \quad \ln \frac{p}{P_B} = \frac{t}{\tau},$$

whence on the one hand

$$t = 2.303 \tau \log \frac{p}{P_B} \quad (39)$$

and for the end of burning

$$t_K = 2.303 \tau \log \frac{P_m}{P_B}; \quad (40)$$

on the other hand

$$\frac{p}{p_B} = e^{\frac{t}{\tau}} \quad \text{or} \quad p = p_B e^{\frac{t}{\tau}}, \quad (41)$$

and for the end of burning

$$p_m = p_B e^{\frac{t_K}{\tau}}. \quad (42)$$

Thus, all the required relationships are derived:

$$p = p_B e^{\frac{t}{\tau}};$$

$$t_K = 2.303 \tau \log \frac{p_m}{p_B};$$

$$\frac{dp}{dt} = \frac{p_m - p_B}{I_K} p = \frac{f \Delta}{1 - \alpha \Delta} \frac{u_1}{e_1} p;$$

$$\tau = \frac{I_K}{p_m - p_B} = \frac{e_1}{u_1} \frac{1 - \alpha \Delta}{f \Delta} [\text{sec}].$$

The magnitude $\tau = \frac{e_1}{u_1(p_m - p_B)} = \frac{e_1}{u_m}$ constitutes the burning time of the powder if it burned at constant pressure $p_m - p_B$ throughout.

The full time of burning at a given loading density is proportional to τ and $\log p_m/p_B$, in other words, it is directly proportional to the

thickness of the powder and inversely proportional to the energy of the powder f and rate of burning u_1 , and decreases with the increase of Δ and p_m (since the value of p_m in the denominator of τ is more effective than in the numerator under the logarithm); it also decreases with the increase of the igniter pressure p_B .

For tubular powder the relation $p = f(t)$ serves as a characteristic curve, whose slope angle ($dp/dt = \tan \varphi$) must continuously increase in proportion to the gas pressure during the entire combustion process, the rate of increase being the higher, the greater are the values of f , Δ , u_1 and the smaller the powder thickness e_1 . At the end of burning the slope angle must be maximum (see figs. 39-42).

We shall derive the relation for the powder gas pressure impulse on the basis of formula (41)

$$\begin{aligned} \int_0^t p dt &= p_B \int_0^t e^{\frac{t}{\tau}} dt = p_B \tau \int_0^{\frac{t}{\tau}} e^{\frac{t}{\tau}} d \frac{t}{\tau} = \\ &= p_B \tau (e^{\frac{t}{\tau}} - 1) = \tau (p - p_B) = \frac{e_1}{u_1} \frac{p - p_B}{p_m - p_B}. \end{aligned} \quad (43)$$

For the end of burning $p = p_m$ and

$$\int_0^{t_K} p dt = \frac{e_1}{u_1} = I_K. \quad (44)$$

The derived formulas confirm the general laws of powder burning

and show that the pressure, the time for complete combustion and the rate of pressure increase depend on the ballistic characteristics and density of loading. Thus, it is precisely the ballistic characteristics f, α, u_1 , the shape and dimensions of the powder (κ, ε, e_1) and the loading density Δ that can be used to control the magnitude and rate of pressure increase of the gases evolved during the burning of powder in a constant volume and to regulate the phenomenon of powder burning and gas formation.

Example. Determine the maximum pressure and time of burning of tubular powder ($2e_1 = 1.00$ mm) when $\Delta = 0.25$;

$$f = 900,000 \text{ kg-dm/kg}; \quad \alpha = 1.00 \text{ dm}^3/\text{kg};$$

$$u_1 = 0.0000075 \text{ dm/sec} : \text{kg/dm}^2, \text{ igniter pressure } p_B = 50 \text{ kg/cm}^2;$$

$$p_m = p_B + \frac{f\Delta}{1 - \alpha\Delta} = 5000 + \frac{900000 \cdot 0.25}{1 - 1.0 \cdot 0.25} = 305,000 \text{ kg/dm}^2 = 3050 \text{ kg/cm}^2;$$

$$I_K = \frac{e_1}{u_1} = \frac{0.005}{0.0000075} = 667 \text{ kg/dm}^2 \cdot \text{sec},$$

$$\tau = \frac{I_K}{p_m - p_B} = \frac{667}{300000} = 0.002223 \text{ sec};$$

$$t_K = 2.303 \tau \log \frac{p_m}{p_B} = 2.303 \cdot 0.002223 \log \frac{3050}{50} = 2.303 \cdot 0.002223 \cdot$$

$$\cdot 1.7853 = 0.00913 \text{ sec.}$$

Curves $p = p_B e^{\frac{t}{\tau}}$ for the given powder will have the following form:

a) at different Δ and the same p_B (fig. 40)

$$\int_0^{t_K'} p dt = \int_0^{t_K''} p dt = \int_0^{t_K'''} p dt$$

b) at the same Δ and different p_B (fig. 41)

$$p_m' - p_m'' = p_B' - p_B''; p_m' - p_m''' = p_B' - p_B'''$$

$$\int_0^{t_K'} p dt = \int_0^{t_K''} p dt = \int_0^{t_K'''} p dt.$$



Fig. 40 - p, t Curves at Different Values of Δ and the Same Value of p_B .

Fig. 41 - p, t Curves at Different Values of p_B and the Same Value of Δ .

For powders of the same kind but varying thickness $e_1' < e_1'' < e_1'''$ at the same loading density and the same igniter pressure p_B , curves p, t will have the form shown in fig. 42, where

$$t_K' : t_K'' : t_K''' = e_1' : e_1'' : e_1''' .$$

When burning powder in small bombs of limited elongation (ratio between length of bomb and its diameter not exceeding 2-3), the curves of pressure increase are gradual in character.

If, however, the powder is burned in a long bomb (1m long by 22mm in dia.) with the powder concentrated at one of its ends, the pressure increase recorded by a meter takes the shape of a wave. In the case of thin powders this condition obtains at relatively low loading densities, of the order of 0.05-0.075, and in the case of thick powders, when Δ is of the order of 0.20-0.25. As the loading density is increased, the growth of the pressure recorder by the crusher increases also: thus, at $\Delta \approx 0.20$, for powders ~ 0.3 mm thick, the maximum pressure varied from 2200-2300 to 7500 kg/cm²; in the case of thick powders at the same value of Δ the pressure was found to be ~ 4000 kg/cm².



GRAPHIC NOT REPRODUCIBLE

Fig. 42 - p, t Curves at Various Values of e_1 and a Constant Value of Δ .

If the crusher cones are placed at the ends of a long bomb and

the powder is ignited from the side at one of the ends, the curves of the pressure growth at both ends will appear wave-like in form if the charge is not distributed uniformly, whereby the maximum of one curve will correspond to the minimum of the other at the same instant of time. These tests show that a wave-like process of pressure distribution occurs in a bomb, where the pressure waves are reflected from one end of the closed pipe to the other.

All of the tests described above were conducted by Vieille in a special long bomb, using cylindrical crushers for recording the pressure growth at both ends. We had verified Vieille's deductions on a similar test set-up using conical crushers.

When the charge concentrated at one end of the bomb is ignited, a localized pressure increase occurs due to gases becoming separated or detached from the burning surface. This pressure increase is the greater, the larger the surface area of the charge, i.e., the thinner the powder. Due to the action of the localized pressure, the gases begin to move to the opposite end of the bomb in the form of a stream whose rate of flow increases rapidly, thus tending to form a vacuum at the point of the start of burning. Upon reaching the opposite end, the gas stream will stop abruptly at its forward end and its kinetic energy will be expended in compressing this part of the bomb until the pressure developed in it exceeds the pressure at the place occupied by the charge. The movement of the stream will then be reversed and the burning of the charge will proceed more intensely under the pressure of the gas stream, creating once again a localized pressure increase. This phenomenon will then be repeated.

At the end of burning of the charge the phenomenon takes on the character of a damped oscillating gas motion inside the bomb.

Similar conditions may occur in the bore of a gun barrel. Therefore, when the loading density is low and the chamber is not completely filled with powder, it is recommended to uniformly distribute the charge along the entire length of the chamber.